Competition in Two Sided Market with Congestion*

Swapnil Sharma

Abstract

Two-sided platforms must balance the benefits created by participation on the opposite side against congestion within each side. This paper studies price competition between two platforms when same-side participation creates congestion and opposite-side participation both generates a network benefit and relieves that congestion. The model adds a balance-dependent congestion term to a Hotelling framework and compares two homing structures. Congestion increases effective platform differentiation but also strengthens the effective cross-group externality, with opposing effects on prices and profits. When both groups single-home and the market-sharing condition holds, the model has a unique symmetric equilibrium, and congestion raises platform profit if and only if the tolerance parameter is below one. In the competitive-bottleneck case, the single-homing side is priced at or below cost and the multi-homing side receives no surplus. Subject to the equilibrium restrictions, congestion raises the maximum symmetric-equilibrium profit if and only if the tolerance parameter exceeds two. The effect of congestion on profit therefore depends on agents' homing choices and on how strongly opposite-side participation offsets same-side crowding.

*I sincerely thank Prof. Rupayan Pal for his useful and insightful suggestions throughout the development of the project. I thank the other faculty members at IGIDR for their guidance and valuable feedback. I also thank Sumit Shrivastva for useful discussions and the participants in the IGIDR Dissertation Workshop for helpful comments.

1 Introduction

Two-sided platforms must attract two groups whose participation decisions are linked. Riders value a ride-hailing service with more drivers, while drivers value access to more riders. Yet participation within a group can also make the service less attractive. A rider may wait longer when rider demand grows faster than driver supply, and a driver may receive fewer bookings when driver supply grows faster than rider demand. Similar imbalances arise on dating, rental, payment, and content platforms. The pricing problem therefore depends on both the size and the composition of a platform's user base.

This paper asks whether congestion can increase a platform's profit and whether the answer changes when users can join more than one platform. The question is not straightforward. Congestion lowers users' utility, but it may also soften price competition by making platforms effectively more differentiated. At the same time, a platform may have to cut the price charged to one group because attracting that group creates more value for the other. The net effect depends on how the two sides interact and on whether they single-home or multi-home.

The model adds congestion to Armstrong's (2006) framework of competing two-sided platforms. A group-i agent on a platform incurs the term $-\beta_i(n_i-\lambda n_j)$, where n_i is participation by the agent's own group and n_j is participation by the other group. The parameter β_i measures sensitivity to same-side crowding. The parameter λ determines how strongly opposite-side participation offsets that crowding. This is a balance-dependent specification rather than a fixed-capacity constraint: when $n_i > \lambda n_j$, the agent bears a congestion cost; when $n_i < \lambda n_j$, the same linear term contributes positively to utility. A larger λ therefore gives opposite-side participation a greater role in relieving congestion.

The specification produces two opposing changes in the pricing problem. Same-side congestion raises effective differentiation from t_i to $t_i + \beta_i$, which tends to increase the price a platform can charge. Congestion relief raises the effective cross-group benefit from α_i to $\alpha_i + \lambda\beta_i$, which tends to reduce that price because an additional user creates more value for the other side. The comparison between these two changes drives the results.

When both groups single-home and the market-sharing condition holds, the model has a unique symmetric equilibrium. Congestion raises each platform's profit relative to the benchmark without congestion if and only if $\lambda < 1$. Below this threshold, the increase in effective differentiation is larger than the increase in the cross-group effect. The model also shows how unequal congestion sensitivities change the price structure: the side that is more sensitive to congestion may pay a higher price while the other side receives a lower price, even when total platform profit rises.

The competitive-bottleneck case has a different pricing logic. One group single-homes and the other multi-homes. Subject to the equilibrium restrictions imposed in the paper, the single-homing side is priced at or below cost, while the multi-homing side is left with no surplus. There is a range of symmetric equilibrium prices for the single-homing side, so the paper compares the maximum equilibrium profit with its no-congestion counterpart. Congestion raises this maximum profit if and only if $\lambda > 2$. A higher value of λ increases the surplus that can be extracted from the multi-homing side. This threshold is attainable only when the other parameter restrictions support the competitive-bottleneck equilibrium; in particular, the model requires sufficiently strong differentiation on the single-homing side.

The paper's contribution is to show how balance-dependent congestion changes the familiar pricing forces in a two-sided market and why homing choices can reverse its effect on profit. The analysis is deliberately static and uses a linear congestion term, allowing the pricing mechanisms and threshold conditions to be stated in closed form. Section 2 reviews the related literature.

Section 3 studies competition when both groups single-home. Section 4 considers a competitive bottleneck with one single-homing group and one multi-homing group. Section 5 concludes.

2 Literature Review

The theory of two-sided markets begins from the observation that a platform chooses a price structure, not only an aggregate price level. Rochet and Tirole (2003) show that the allocation of charges across the two sides affects participation, transaction volume, and surplus when each side values access to the other. Rochet and Tirole (2006) sharpen the definition of two-sidedness by asking whether the volume of transactions changes when the price structure changes while the total price is held fixed. Caillaud and Jullien (2003) emphasize the coordination problem created by indirect network effects. Their analysis shows how subsidies, non-exclusive access, and usage-based discrimination can help an intermediary attract both sides of the market.

Armstrong (2006) provides the benchmark used in this paper. His monopoly, single-homing duopoly, and competitive-bottleneck models identify three determinants of equilibrium prices: the strength of cross-group externalities, the use of membership or transaction fees, and agents' homing decisions. Platform differentiation also matters because it governs the elasticity of participation. Armstrong and Wright (2007) allow differentiation to vary across the two sides and show how competitive bottlenecks can arise endogenously. When sellers view platforms as close substitutes but buyers have stronger platform preferences, platforms subsidize buyers and extract surplus from multi-homing sellers. The present paper retains this distinction between single-homing competition and a competitive bottleneck, but asks how congestion changes the relevant price-cost margins.

Subsequent work adds pricing factors that are held fixed here. Hagiu (2006) shows that the timing of participation and a platform's ability to commit to future prices affect exclusive participation and multi-homing equilibria. Weyl (2010) demonstrates that optimal platform prices depend on heterogeneity within each side because a profit-maximizing platform internalizes the network benefits created for marginal users rather than for all users. Hagiu (2009) makes the cross-group effects endogenous through buyers' preference for product variety and competition among sellers. In his model, variety, seller market power, and economies of scale in multi-homing alter which side bears a larger share of platform charges. These papers clarify why a pricing rule based only on the average cross-group externality can be incomplete.

A related literature studies negative effects within a side of the platform. Belleflamme and Toulemonde (2009) combine positive cross-group externalities with negative intra-group externalities and examine whether a profit-seeking entrant can displace a free incumbent platform. They show that entry may be impossible when the negative intra-group effect is sufficiently strong relative to the positive cross-group effect. Hagiu (2009) and Belleflamme and Peitz (2019) model seller competition more explicitly. Seller participation increases variety for buyers but also intensifies competition among sellers, so a platform may prefer to limit seller entry. These models interpret the negative same-side effect mainly as business stealing or reduced seller margins. The congestion term in the present paper applies symmetrically to both groups and depends on participation on the opposite side as well as participation within the group.

The papers closest to the present pricing model are Aloui and Jebsi (2010, 2011). Aloui and Jebsi (2010) introduce one-sided congestion into a monopoly platform and show that the platform allocates the marginal congestion cost across both sides. Aloui and Jebsi (2011) extend Armstrong's Hotelling duopoly to congestion on both sides. Congestion softens competition for market share and modifies the divide-and-conquer pricing rule: the difference between the two

sides' prices depends on their marginal congestion costs. In those papers, congestion is generated by the number of same-side users relative to a capacity constraint. Here, an agent's congestion cost depends on the excess of same-side participation over a tolerated proportion of opposite-side participation. This specification allows an additional participant on the other side to create a direct network benefit and to relieve congestion at the same time. The tolerance parameter therefore changes both the effective network effect and the effective differentiation between platforms.

Much of the applied congestion literature concerns broadband networks. Choi and Kim (2010) and Cheng, Bandyopadhyay, and Guo (2011) study paid priority and capacity investment under departures from network neutrality. Both show that prioritization changes how surplus is divided between an internet service provider and content providers, while its investment and welfare effects depend on the details of capacity and demand. Economides and Tåg (2012) isolate the two-sided pricing issue by studying fees to users and content providers without modeling traffic priority. They find that a zero fee to content providers can raise total surplus for some parameter values when broadband access is monopolized.

Krämer and Wiewiorra (2012) allow content providers to differ in their sensitivity to congestion. Quality-of-service tiering can allocate scarce capacity toward providers that value priority more and can strengthen long-run investment incentives, although the preferred regime depends on the distribution of congestion sensitivity. Bourreau, Kourandi, and Valletti (2015) extend the net-neutrality analysis to competing internet platforms. Wang, Ma, and Xu (2017) model endogenous congestion in a network that charges end users and content providers. They show that profit- and welfare-maximizing prices depend on throughput elasticity, capacity, congestion sensitivity, and the type of traffic. This work treats congestion as a capacity or queuing problem. The present model instead represents crowding as a participation externality and studies how it interacts with homing choices.

Recent work broadens the meaning of congestion beyond physical capacity. Calsamiglia, Doval, Robinson-Cortes, and Shum (2023) study an online rental platform where a common ranking directs many renters toward the same small set of rooms. Their estimates show that ranking design can create avoidable matching congestion and reduce the number of completed matches. Chen and Sheldon (2016) provide a complementary supply response in ride-hailing: drivers respond to surge pricing, so price can help rebalance the two sides when demand is high. These studies make platform design and dynamic supply adjustment central. The model here is static and uses congestion sensitivity and tolerance as primitives, which makes it possible to derive closed-form comparisons between single-homing and multi-homing environments.

The contribution of this paper is therefore narrower than a general theory of congestion management. It adds a balance-dependent congestion term to Armstrong's pricing framework and traces its effect through the same two mechanisms that drive the benchmark: differentiation and cross-group feedback. This formulation produces simple thresholds for when congestion raises platform profit and shows why the threshold changes when one side multi-homes.

3 Competition in Two Sided single-homing

There are two competing platforms and here we assume that each agent choose to join single platform.

Suppose there are two group of agent, say 1 and 2 and two platforms, say A and B. Platform A and B allow the two groups to interact. A member in each group cares about both - the number of agents of other group and also the number of agents of their own group on the platform. An

Agent in any group derive a positive utility from the presence of agents of the other group and negative utility from the agents of their own group. If agents of group 1 and 2 join the platform A, they will get (u_1^A, u_2^A). If platform A attracts n_1^A and n_2^A agents of the two groups, then the utilities they derive are -

$$u_1^A = \alpha_1 n_2^A - p_1^A - \beta_1(n_1^A - \lambda n_2^A)$$
$$u_2^A = \alpha_2 n_1^A - p_2^A - \beta_2(n_2^A - \lambda n_1^A)$$

where p_1^A and p_2^A are the prices charged by the platform A to the two groups. The parameter α_1 measures the benefit a group-1 agents from interacting with each group-2 agent, and α_2 measures the benefit a group-2 agents from interacting with each group-1 agent. The parameter β_1 is the measure for loss in the utility due to congestion i.e. as the n_1 increases from λ times n_2 by one unit, each agent of group-1 is getting a dis utility which is measured by β_1. Similar interpretation is for β_2. The parameter β is known as congestion parameter. λ is a tolerance or threshold parameter which can be interpreted as- for a given n_1^A and n_2^A in equation (1), as λ increases dis-utility from congestion reduces. In this sense we can say λ is the parameter which measure the tolerance of each agent for congestion. Here we assume tolerance level for both groups is same. After rearranging the above expressions we get,

$$u_1^A = (\alpha_1 + \beta_1\lambda)n_2^A - p_1^A - \beta_1 n_1^A \tag{1}$$

$$u_2^A = (\alpha_2 + \beta_2\lambda)n_1^A - p_2^A - \beta_2 n_2^A \tag{2}$$

Similarly, for the agents who join platform B

$$u_1^B = (\alpha_1 + \beta_1\lambda)n_2^B - p_1^B - \beta_1 n_1^B \tag{3}$$

$$u_2^B = (\alpha_2 + \beta_2\lambda)n_1^B - p_2^B - \beta_2 n_2^B \tag{4}$$

Utility offered by the platforms A and B to an agent of group 1 are (u_1^A, u_1^B) and similarly for an agent of group 2 - (u_2^A, u_2^B). Now assume the number of agents who join the platform A is given by the Hotelling specification in which, it is assume that the platforms are located at two endpoints and agents are uniformly located on $[0, 1]$.

$$n_1^A = \frac{1}{2} + \frac{u_1^A - u_1^B}{2t_1}; \qquad n_2^A = \frac{1}{2} + \frac{u_2^A - u_2^B}{2t_2} \tag{5}$$

where t_1, t_2 are the product differentiation parameters for the two groups that describe the competitiveness of two sides of the market. It is similar to transportation cost in Hotelling problem. In that sense another possible interpretation for this is costs of signing up for a service, initial set up costs for learning about a new service or cost of reaching to the platforms.

Now using Equation (1), (2), (3), (4) in equation (5) and the fact that $n_1^B = 1 - n_1^A$, we get following expression -

$$\begin{aligned} n_1^A =& \frac{1}{2} + \frac{(\alpha_1 + \beta_1\lambda)(2n_2^A - 1) - (p_1^A - p_1^B) - \beta_1(2n_1^A - 1)}{2t_1}; \\ n_2^A =& \frac{1}{2} + \frac{(\alpha_2 + \beta_2\lambda)(2n_1^A - 1) - (p_2^A - p_2^B) - \beta_1(2n_2^A - 1)}{2t_2} \end{aligned} \tag{6}$$

We can further simplifies the equation (6) to get some important interpretation -

$$\begin{aligned} n_1^A =& \frac{1}{2} + \frac{(\alpha_1 + \lambda\beta_1)(2n_2^A - 1) - (p_1^A - p_1^B)}{2(t_1 + \beta_1)}; \\ n_2^A =& \frac{1}{2} + \frac{(\alpha_2 + \beta_2\lambda)(2n_1^A - 1) - (p_2^A - p_2^B)}{2(t_2 + \beta_2)} \end{aligned} \tag{7}$$

From equation (7) we can easily see that, one extra agent of group-2 on the platform will attract $\frac{(\alpha_1+\beta_1\lambda)}{(t_1+\beta_1)}$ agents of group-1 on the platform. Also if we compare equation (7) with the case of no congestion (Armstrong model) i.e. β_1 and β_2 equal to 0, we get that the market power of platform increased from t_1,t_2 to $(t_1 + \beta_1)$,$(t_2 + \beta_2)$ respectively and network effect increased from α_1, α_2 to $(\alpha_1 + \lambda\beta_1)$,$(\alpha_2 + \lambda\beta_2)$ respectively. This interpretation will give us some nice insight to explain the some results in the end of this section.

Given that platforms will charge the price pairs (p_1^A, p_2^A),(p_1^B, p_2^B), solving the equation (6) or (7) simultaneously, we get

$$\begin{aligned} n_1^A =& \frac{1}{2} + \frac{(\alpha_1 + \lambda\beta_1)(p_2^B - p_2^A) + (t_2 + \beta_2)(p_1^B - p_1^A)}{2[(t_1 + \beta_1)(t_2 + \beta_2) - (\alpha_1 + \beta_1\lambda)(\alpha_2 + \beta_2\lambda)]}; \\ n_2^A =& \frac{1}{2} + \frac{(\alpha_2 + \lambda\beta_2)(p_1^B - p_1^A) + (t_1 + \beta_1)(p_2^B - p_2^A)}{2[(t_1 + \beta_1)(t_2 + \beta_2) - (\alpha_1 + \beta_1\lambda)(\alpha_2 + \beta_2\lambda)]} \end{aligned} \tag{8}$$

Let us assume that each platform has a per-agent cost f_1 and f_2 for serving group-1 and group-2 respectively. So the profit of platform A's is

$$\begin{aligned} \pi^A = (p_1^A - f_1)\left[\frac{1}{2} + \frac{(\alpha_1 + \lambda\beta_1)(p_2^B - p_2^A) + (t_2 + \beta_2)(p_1^B - p_1^A)}{2[(t_1 + \beta_1)(t_2 + \beta_2) - (\alpha_1 + \beta_1\lambda)(\alpha_2 + \beta_2\lambda)]}\right] + \\ (p_2^A - f_2)\left[\frac{1}{2} + \frac{(\alpha_2 + \lambda\beta_2)(p_1^B - p_1^A) + (t_1 + \beta_1)(p_2^B - p_2^A)}{2[(t_1 + \beta_1)(t_2 + \beta_2) - (\alpha_1 + \beta_1\lambda)(\alpha_2 + \beta_2\lambda)]}\right] \end{aligned} \tag{9}$$

Now before going into the final expression for profit maximizing price pair, we should derive the expression for market share equilibria to exist. The necessary and sufficient condition for market sharing to exist is

$$4(t_1 + \beta_1)(t_2 + \beta_2) > ((\alpha_1 + \alpha_2) + \lambda(\beta_1 + \beta_2))^2 \tag{10}$$

This expression can be derive from stability condition i.e. determinant of Hessian matrix is greater than zero.

From expression (9), we can find the best response function of platform A's to platform B's price pair through first order condition. From expression (10), we can see that there is no asymmetry. Thus for the case of symmetric equilibrium where each platform offers the same price pair (p_1, p_2), the first order condition for equilibrium price are

$$\begin{aligned} p_1 =& f_1 + (t_1 + \beta_1) - \frac{(\alpha_2 + \lambda\beta_2)}{(t_2 + \beta_2)}(\alpha_1 + \lambda\beta_1 + p_2 - f_2); \\ p_2 =& f_2 + (t_2 + \beta_2) - \frac{(\alpha_1 + \lambda\beta_1)}{(t_1 + \beta_1)}(\alpha_2 + \lambda\beta_2 + p_1 - f_1) \end{aligned} \tag{11}$$

If there is no cross group externalities or congestion, then in Hotelling model the equilibrium price would be $p_1 = f_1 + t_1$. Thus all the term other than f_1 and t_1 is due to cross group externalities with congestion.

Now on solving the equation (11) simultaneously, we get

Lemma 1. Suppose equation (10) hold. Then the congestion model with two sided single homing has a unique equilibrium and that is symmetric. Equilibrium price for group-1 and group-2 are given by

$$\begin{aligned} p_1 =& f_1 + (t_1 + \beta_1) - (\alpha_2 + \lambda\beta_2); \\ p_2 =& f_2 + (t_2 + \beta_2) - (\alpha_1 + \lambda\beta_1) \end{aligned} \tag{12}$$

Let the equilibrium price level for group-1 and group-2 in case of no congestion is denoted by p_1^o and p_2^o respectively. This is obtained by putting congestion parameters, β_1 and β_2, equal to zero. Obtained expression is

$$\begin{aligned} p_1^o =& f_1 + t_1 - \alpha_2; \\ p_2^o =& f_2 + t_2 - \alpha_1 \end{aligned} \tag{13}$$

This is same as the equilibrium price level in Armstrong (2006) paper. On comparing equation equation (12) and (13), we can say that transportation cost is increased from (t_1, t_2) to $((t_1 + \beta_1), (t_2 + \beta_2))$ for both groups because of congestion. Similarly network parameter increase from (α_1, α_2) to $((\alpha_1 + \lambda\beta_1), (\alpha_2 + \lambda\beta_2))$ in case of congestion. Increased in transportation cost in case of congestion is because now when an agent of group i join a platform will not only incured a transportation cost t_i but also gets a dis-utility β_i due to congestion. So the effective transportation will become $(t_i + \beta_i)$. This increased in transportation cost will give more power to platform to set a higher prices. Increase in network effect is due to the fact that in case of congestion when an agent of group j joins a platform it will not only increased the utility of opposite group by α_i but also decrease the dis-utility because of congestion by $\lambda\beta_i$. Thus the effective network effect becomes $(\alpha_i + \lambda\beta_i)$. This increase in network effect reduce the power the platform to set higher prices.

From (12), in equilibrium each platform makes profit

$$\pi^* = \frac{(t_1 + \beta_1) + (t_2 + \beta_2) - (\alpha_1 + \lambda\beta_1) - (\alpha_2 + \lambda\beta_2)}{2}$$

Now the profit expression in case of no congestion (Armstrong's Model) i.e. β_1 and β_2 equal to zero is given by

$$\pi^o = \frac{(t_1 + t_2) - (\alpha_1 + \alpha_2)}{2}$$

So the condition when congestion is beneficial (fetch more profit) for the platform is given by - $(\pi^* - \pi^o) > 0$.

Proposition 1. Suppose condition (10) holds and equilibrium price for both the groups is given by equation (12). Then condition under which platforms earn higher profit because of congestion is -

$$\lambda < 1$$

It means that the when tolerance parameter is less than one, it is profitable for the platform.

This result is quiet intuitive also. As we have seen that the market power of platforms to set higher price because of congestion is increased by β_i(since transportation cost is increased to $(t_i + \beta_i)$ from t_i). And decreased by $\lambda\beta_i$ (network effect increased to $(\alpha_i + \lambda\beta_i)$ from α_i) for group i. Thus the value of λ will determine the overall increase in market power of platform for each group and subsequently the profit for platform. So we will discuss below the various cases that can arise on the possible value of λ. We re-write the equation (12) in following manner -

$$p_1 = f_1 + t_1 - \alpha_2 + (\beta_1 - \lambda\beta_2);$$
$$p_2 = f_2 + t_2 - \alpha_1 + (\beta_2 - \lambda\beta_1)$$

On comparing the above expression with equation (13), it can easily be seen that $p_1 > p_1^o$ whenever $(\beta_1 - \lambda\beta_2) > 0$ or $\lambda < (\frac{\beta_1}{\beta_2})$. It means when $\lambda < (\frac{\beta_1}{\beta_2})$, the equilibrium price charged by the platforms to group 1 is greater in case of congestion. Similarly when $(\beta_2 - \lambda\beta_1) > 0$ or $\lambda < (\frac{\beta_2}{\beta_1})$ equilibrium price charged from group 2 by the platform is higher in case of congestion.

Case 1. when $\lambda < min\left[\left(\frac{\beta_1}{\beta_2}\right), \left(\frac{\beta_2}{\beta_1}\right)\right]$

In this case price charged by both the group is higher because of congestion (i.e. $p_1 > p_1^o$ and $p_2 > p_2^o$). And since $\lambda < min\left[\left(\frac{\beta_1}{\beta_2}\right), \left(\frac{\beta_2}{\beta_1}\right)\right]$, it means λ is less than one also. Thus the platform earns more profit due to congestion ($\pi^* > \pi^o$). This is because, the increment in the transportation cost (from t_i to $(t_i + \beta_i)$) is higher than the increase in network effect (which increase from α_i to $(\alpha_i + \lambda\beta_i)$. Due to which platforms can set higher price for both the group and earn higher profits in the equilibrium.

Case 2. when $\lambda > max\left[\left(\frac{\beta_1}{\beta_2}\right), \left(\frac{\beta_2}{\beta_1}\right)\right]$

In this case price charged by both the group is lower because of congestion (i.e. $p_1 < p_1^o$ and $p_2 < p_2^o$). And since $\lambda > max\left[\left(\frac{\beta_1}{\beta_2}\right), \left(\frac{\beta_2}{\beta_1}\right)\right]$, it means λ is more than one also. Thus the platform earns lesser profit due to congestion ($\pi^* < \pi^o$). Reasoning is opposite to the case 1.

For other two cases, let us assume without loss of generality that $\beta_1 > \beta_2$.

Case 3. when $\left(\frac{\beta_2}{\beta_1}\right) < \lambda < 1 < \left(\frac{\beta_1}{\beta_2}\right)$

In this case price charged by the group-1 is higher and by the group-2 is lower because of congestion ($p_1 > p_1^o$ and $p_2 < p_2^o$). That is, due to congestion agents of group-2 gets a subsidy at the cost of higher price charge by group-1 (the group which have higher congestion parameter have to pay higher price). Since the λ is less than one, so the overall profit of both the platform is higher because of congestion ($\pi^* > \pi^o$). Platform extract more from group-1 then it transfer to group-2, due to which overall profit of platform increased.

Case 4. when $\left(\frac{\beta_2}{\beta_1}\right) < 1 < \lambda < \left(\frac{\beta_1}{\beta_2}\right)$

This case is similar to the previous case, with only difference is that now the profit of both the platform is lower with congestion in comparison to without congestion. i.e $p_1 > p_1^o$ and $p_2 < p_2^o$ but $\pi^* < \pi^o$. Reasoning is similar to case 3, only difference here is that the platform can not extract more from group-1 then what it transfer to group-2, due to which overall profit of platform decreases.

Remark: This is a static model and it will be difficult to capture dynamics of the congestion through this model. But still we can infer some information which is dynamic in nature and have some policy implication. Suppose tolerance parameter is greater than one initially. Then it is in benefit of platform to reduce the tolerance parameter (and hence tolerance) of the agents on the platform. The only way for this is to give better and innovative service to all the agents. Then over time, tolerance parameter (and hence tolerance) will reduce and platform start earning more profit by congestion. Thus congestion will leads to better service and innovation. This is true for net-neutrality principle also. By abandoning net-neutrality principle, platform start getting more profit immediately and there is very less effect of congestion on pricing and profit of the platform. Thus abandoning of net-neutrality principle, can seriously hamper the innovation and urge to provide better service to the agents on the platform.

4 Competitive Bottlenecks: Multi-homing case

This section represent the case when multi-homing is possible. Multi-homing means the an agent join both the platforms. Now there are two cases to consider: (i) one group of agents are single home and other group is multi-home, and (ii) both the groups are multi-homes. If the main purpose of joining the platform is to derive a utility by interacting the opposite group then case (ii) will not arise, so only case (i) is discussed here. This is because, if the opposite group is already a multi-homes, then the agents of single homing group have no incentive to join the next platform. Thus the group which is multi-home should not get any intrinsic utility just by joining any platform and agents of single homing group get intrinsic utility only once (duplicate intrinsic utility is not allowed). Before going into the details of multi-homing, first we need to see the condition under which a group is always single homing. An agent will join only one platform when the cost of joining other platform is higher than the utility it gets from joining another platform. The cost of joining the another platform for an agent of group i with congestion is $(t_i+\beta_i)$ (effective transportation cost in case of congestion) plus price charged by the platform where as the utility

it gets by joining another platform is given by $(\alpha_i + \lambda\beta_i)$. So whenever the price of joining a new platform is non-negative and $(t_i + \beta_i) > (\alpha_i + \lambda\beta_i)$, agent will not join the another platform. Price have to be non-negative for sufficient condition because in case when transportation cost is higher that utility derive but if platform pay (i.e negative price) for subscription then it is be possible that net utility becomes higher by joining one more platform. Non-negative restriction on price avoid that case.

Without loss of generality we assume that agents of group-1 is single homing and group-2 agents are multi-home. Thus for group-1 $(t_1 + \beta_1) > (\alpha_1 + \lambda\beta_1)$ with non-negative price restriction must holds. For agents of group-2, transportation cost must less than network effect. For simplicity it is assume that $t_2 = 0$. Also the non-negative restriction of price charged is put on the platform for both the groups. Thus to model the multi-homing case, we make following assumptions:

A1. Intrinsic utility of joining the platform for agents of group-1 (let say u_1^o) is sufficiently high and intrinsic utility of group-2 agents (let say u_2^o) by joining any platform is zero. This assumption ensures that all the agents of group-1 will join at least one platform and agents of group-2 will only join any platform in case when net utility becomes higher by joining the platform.

A2. Price charged by the platforms from both the group must be non-negative.

A3. $(t_1 + \beta_1) > (\alpha_1 + \lambda\beta_1)$. A2 and A3 ensure that the group-1 is single homing. A3 can also be written in this form $\lambda < 1 + \left(\frac{t_1 - \alpha_1}{\beta_1}\right)$

A4. $t_2 = 0$. This assumption ensure that every agent of group-2 will either join a platform or will not join. It will not be the case that some agents of group-2 join a given platform and some will not. This is because when transportation cost (t_2) is zero, agents become homogeneous i.e. there is no difference within the group.

A5. $f_2 < \frac{t_1}{2}$. There are two benefits of platforms to serve the agents of group-2. First, it will increase the revenue for the platforms. Second, it enable the platforms to compete better for agents of group-1. The benefit of joining an agent of group-2 must outweigh the cost of serving the agent. A5 ensure this condition.

If assumption A1 to A5 holds, then the agents of group-1 will always be single homing and there will be four possible configuration for group-2 to consider: (i) all agents of group-2 will multi-home, (ii) all agents of group-2 will single home on platform A, (iii) all agents of group-2 will single home on platform B, and (iv) all agents of group-2 do not join any platform. Here we assume that, when an agent of group-2 is indifferent between joining or not joining any platform then (s)he will join the platform.

Configuration 1. All agents of group-2 will multi-home

Since every agents of group-2 join both the platform, therefore $n_2^A = n_2^B = 1$. And the utility of an agent of group-1 by joining platform A is given by-

$$u_1^A = \alpha_1(n_2^A) - p_1^A - \beta_1(n_1^A - \lambda(n_2^A)) + u_1^o$$

$$u_1^A = \alpha_1(1) - p_1^A - \beta_1(n_1^A - \lambda(1)) + u_1^o$$

Above can also be written in following manner:

$$u_1^A = (\alpha_1 + \beta_1\lambda) - p_1^A - \beta_1 n_1^A + u_1^o$$

Similarly,

$$u_1^B = (\alpha_1 + \beta_1\lambda) - p_1^B - \beta_1 n_1^B + u_1^o$$

As in the previous section, we assume that the platforms are located at two endpoint of and agent are distributed uniformly on $[0, 1]$. Also let us assume that the number of agents of group-1 on any platform is given by Hotelling specification. Thus the number of agents of group-1 who will join the platform A and B is given by

$$\begin{aligned} n_1^A =& \frac{1}{2} + \frac{p_1^B - p_1^A}{2(t_1 + \beta_1)}; \\ n_1^B =& \frac{1}{2} + \frac{p_1^A - p_1^B}{2(t_1 + \beta_1)} \end{aligned} \tag{14}$$

where (p_1^A, p_1^B) have the same meaning as in Section 3.

Now, the utility of an agent of group 2 (who joins both the platform) is given by -

$$u_2^{AB} = \alpha_2(n_1^A + n_1^B) - p_2^A - p_2^B - \beta_2(n_2^A - \lambda n_1^A) - \beta_2(n_2^B - \lambda n_1^B)$$

$$u_2^{AB} = \alpha_2 - p_2^A - p_2^B - \beta_2(2 - \lambda)$$

Agents of group-2 will multi-home if following conditions holds:

Condition 1. Agents of group-2 should prefer multi-homing in comparison to not joining any platform. That is, the utility derive by the group-2 agents by multi-homing must be non-negative.

$$u_2^{AB} = \alpha_2 - p_2^A - p_2^B - \beta_2(2 - \lambda) \geqslant 0$$

$$p_2^A + p_2^B \leqslant (\alpha_2 + \beta_2\lambda) - 2\beta_2$$

Condition 2. Group-2 agents prefer multi-homing than to single homing at platform A. This will happens only when the agents of group-2 gets utility higher in case of multi-homing.

$$\alpha_2 - p_2^A - p_2^B - \beta_2(2 - \lambda) \geqslant \alpha_2(n_1^A) - p_2^A - \beta_2(1 - \lambda n_1^A)$$

$$p_2^B \leqslant (\alpha_2 + \beta_2\lambda)n_1^B - \beta_2 \tag{15}$$

Condition 3. Group-2 agents prefer multi-homing than to single homing at platform B. Same as above, utility in case of multi-homing should be higher (or equal) in case of multi-homing.

$$\alpha_2 - p_2^A - p_2^B - \beta_2(2 - \lambda) \geqslant \alpha_2(n_1^B) - p_2^B - \beta_2(1 - \lambda n_1^B)$$

$$p_2^A \leqslant (\alpha_2 + \beta_2\lambda)n_1^A - \beta_2 \tag{16}$$

We can easily check that, condition 1 automatically satisfy when condition 2 and 3 hold. The profits of the platforms' are:

$$\pi^A = (p_1^A - f_1)\left[\frac{1}{2} + \frac{p_1^B - p_1^A}{2(t_1 + \beta_1)}\right] + (p_2^A - f_2) \tag{17}$$

$$\pi^B = (p_1^B - f_1)\left[\frac{1}{2} + \frac{p_1^A - p_1^B}{2(t_1 + \beta_1)}\right] + (p_2^B - f_2) \tag{18}$$

Configuration 2. All agents of group-2 will join only platform A

Since group-2 agents only join platform A, therefore $n_2^A = 1; n_2^B = 0$ The proportion of group-1 agents who join the platform A will be given by-

$$n_1^A = \frac{1}{2} + \frac{(p_1^B - p_1^A) + (\alpha_1 + \lambda\beta_1)}{2(t_1 + \beta_1)}$$

All agents of group-2 will join only platform A if following conditions holds:

Condition 1. Group-2 agents prefer to join platform A in comparison to not joining any platform:

$$\alpha_2(n_1^A) - p_2^A - \beta_2(1 - \lambda n_1^A) \geqslant 0$$

$$p_2^A \leqslant (\alpha_2 + \beta_2\lambda)n_1^A - \beta_2$$

Condition 2. Group-2 agents prefer to single home on platform A in comparison to multi-home:

$$\alpha_2(n_1^A) - p_2^A - \beta_2(1 - \lambda n_1^A) \geqslant \alpha_2 - p_2^A - p_2^B - \beta_2(1 - \lambda)$$

$$p_2^B \geqslant (\alpha_2 + \beta_2\lambda)n_1^B$$

Condition 3. Group-2 agents prefer to single home on platform A instead of platform B:

$$\alpha_2(n_1^A) - p_2^A - \beta_2(1 - \lambda n_1^A) \geqslant \alpha_2(1 - n_1^A) - p_2^B - \beta_2(-\lambda n_1^B)$$

$$p_2^B - p_2^A \geqslant (\alpha_2 + \beta_2\lambda)(1 - 2n_1^A) + \beta_2$$

Condition 3 follows from condition 1 and 2. The profits of the platforms' are:

$$\pi^A = (p_1^A - f_1)\left[\frac{1}{2} + \frac{p_1^B - p_1^A + (\alpha_1 + \lambda\beta_1)}{2(t_1 + \beta_1)}\right] + (p_2^A - f_2)$$

$$\pi^B = (p_1^B - f_1)\left[\frac{1}{2} + \frac{p_1^A - p_1^B - (\alpha_1 + \lambda\beta_1)}{2(t_1 + \beta_1)}\right]$$

Configuration 3. All agents of group-2 will join only platform B

By symmetry $n_2^A = 0; n_2^B = 1$ and proportion of group-1 agent on platform B is given by:

$$n_1^B = \frac{1}{2} + \frac{(p_1^A - p_1^B) + (\alpha_1 + \lambda\beta_1)}{2(t_1 + \beta_1)}$$

All agents of group-2 will join only platform B if following conditions holds:

Condition 1. Group-2 agents prefer to join platform B in comparison to not joining any platform:

$$\alpha_2(n_1^B) - p_2^B - \beta_2(1 - \lambda n_1^B) \geqslant 0$$

$$p_2^B \leqslant (\alpha_2 + \beta_2\lambda)n_1^B - \beta_2$$

Condition 2. Group-2 agents prefer to single home on platform B instead of multi-home:

$$\alpha_2(n_1^B) - p_2^B - \beta_2(1 - \lambda n_1^B) \geqslant \alpha_2 - p_2^B - p_2^A - \beta_2(1 - \lambda)$$

$$p_2^A \geqslant (\alpha_2 + \beta_2\lambda)n_1^A$$

Condition 3. Group-2 agents prefer to single home on platform B in comparison to single home on platform A:

$$\alpha_2(n_1^B) - p_2^B - \beta_2(1 - \lambda n_1^B) \geqslant \alpha_2(1 - n_1^B) - p_2^A - \beta_2(-\lambda n_1^A)$$

$$p_2^A - p_2^B \geqslant (\alpha_2 + \beta_2\lambda)(1 - 2n_1^B) + \beta_2$$

The profits of the platforms' are:

$$\pi^A = (p_1^A - f_1)\left[\frac{1}{2} + \frac{p_1^B - p_1^A - (\alpha_1 + \lambda\beta_1)}{2(t_1 + \beta_1)}\right]$$

$$\pi^B = (p_1^B - f_1)\left[\frac{1}{2} + \frac{p_1^A - p_1^B + (\alpha_1 + \lambda\beta_1)}{2(t_1 + \beta_1)}\right] + (p_2^B - f_2)$$

Configuration 4. All agents of group-2 do not join any platform

The proportion of agents of group-1 who join platform A and B are same as equation (14). This configuration is normal Hotelling problem for agents of group-1 where pricing for group-1 is given by -

$$p_1^A = p_1^B = f_1 + t_1 + \beta_1$$

This configuration will happens only when all the inequalities of configuration 1 are reversed.

Remark: For a range of prices; there are more one configuration is possible. For example: suppose both the platform offer same pair of prices (p_1, p_2). If $p_2 \leqslant \frac{(\alpha_2+\lambda\beta_2)}{2} - \beta_2$, then configuration 1 is consistent. If in addition to above condition; p_2 also satisfy the condition $p_2 \geqslant (\alpha_2 + \lambda\beta_2)\Big(\frac{1}{2} -$

$\frac{(\alpha_1+\lambda\beta_1)}{2(t_1+\beta_1)}\Big)$, then configuration 1 and 2 both are consistent. Thus agents have to coordinate their choice for a given price pair.

Now to deal with the issue of multiplicity of consistent demand configuration, we have to impose certain restriction. As in Armstrong and Wright (2007), this paper also impose following two restrictions on the range of possible equilibria.

1. Inertia Condition: This condition states that the equilibrium must be robust to the deviation or change in price set by the platform provided the change in price do not result in change in configuration i.e. the original configuration remain consistent with changed price. If configuration change, then new configuration will require simultaneous coordination of agents to move. So the inertia condition says that if it is difficult for agents to coordinate to move to new configuration, the equilibrium price should be such that no deviation is possible.

2. Monotonicity Condition: According to this condition, if platform deviate from the equilibrium prices such that (weakly) reduces both the prices, then it must (weakly) increase that platform's demand from both the groups.

Given these two conditions (or restrictions), we try to construct the equilibrium pricing for both the platforms. If inertia condition holds the equilibrium price set by both the platforms are such that group-2 left with no surplus.

Proposition 2. Let assumption A1, A3 and A4 holds and suppose that the inertia condition applies. Then in the equilibrium agents of group-2 will left with no surplus.

Proof: Lets prove for configuration-1 and proof for other configuration also goes with similar argument. Let the equilibrium price for group-2 is (p_2^A, p_2^B). Now consider by contradiction that the agents of group-2 left with some surplus in equilibrium. This means that the first condition of configuration-1 does not satisfy with equality (this condition is obtained by the condition $u_2^{AB} \geqslant 0$). This means either inequality (15) or (16) does not satisfy with equality. Thus from the expression (17) and (18), it is possible for the platform to increase the profit until the expression (15) and (16) satisfy with equality. And inertia condition ensure that we remain in the configuration-1. Thus this cannot be equilibrium price. At equilibrium the expression (15) and (16) satisfy with equality and group-2 agents' do not left with any surplus. Through similar argument we can easily prove for configuration 2 and 3.

Now we try to construct a symmetric equilibrium. Thus configuration-2 and 3 is out of picture and only configuration 1 or 4 will apply. But configuration-4 is not of much interest from the view point of two-sided market (in fact pricing mechanism is already mentioned previously). Thus we focus mainly on configuration-1 for symmetric equilibria. By proposition-2, any equilibrium will involves expression (15) and (16) to hold with equality. And in case of symmetric equilibrium price charged by group-2 will be given by -

$$p_2 = p_2^A = p_2^B = \frac{(\alpha_2 + \lambda\beta_2)}{2} - \beta_2$$

or,

$$p_2 = \frac{\alpha_2 + (\lambda - 2)\beta_2}{2}$$

where (p_1, p_2) are symmetric equilibrium price pair set by both the platform. If platform B set the price $(p_1, \left(\frac{\alpha_2+(\lambda-2)\beta_2}{2}\right))$ in equilibrium, then the platform A can deviate if deviation is profitable. Thus if p_1 is equilibrium price for group-1, then it has to be in a manner such that any deviation from p_1 is non-profitable for both the platform.

The fact that the zero surplus is left with group-2 in equilibrium, reduces the possible set of equilibrium significantly. The reason is that suppose platform A undercut the price for group-1 slightly, then by monotonicity condition this slight undercut will attract more agents of group-1 on platform A (and hence fewer on platform B). Since initially the utility of agents of group-2 is zero on both the platform, now slight undercut of price by platform A for group-1 will left negative utility for group-2 on platform B. Thus a small undercut in price for group-1 by the platform A will result that the group-2 join platform A exclusively. That is, small reduction in price of group-1 cause us to move to configuration-2 from configuration-1. But in case of symmetric equilibrium, both the platform undercut the price of subscription for group-1. Thus the reduction in p_1^A or p_1^B is profitable only when the equilibrium p_1 is greater than marginal cost of serving of group-1 agent. That is, deviation is profitable only when $p_1 > f_1$.

Lemma 2. When $\alpha_1, \alpha_2 > 0$, the monotonicity condition holds in addition to assumptions A1, A3 and A4, and any symmetric equilibrium price for the single-homing side is at or below the cost of serving that side, i.e. $p_1 \leqslant f_1$.

Another possible deviation available for platform A is to increase the price of group-1 (p_1^A) and reduce the price for group-2 (p_2^A) such that configuration-1 remain feasible. Given the inertia condition, this deviation will not result in change in configuration. For (p_1, p_2) to equilibrium, there does not exist such deviation which increase the profit of platform A. That is, derivative of profit expression with respect to p_1^A at $(p_1^A = p_1)$ should be non-positive.

$$\begin{aligned}
\pi^A &= (p_1^A - f_1)n_1^A + (p_2^A - f_2) \\
\pi^A &= (p_1^A - f_1)n_1^A + (\alpha_2 + \beta_2\lambda)n_1^A - \beta_2 - f_2 \\
\pi^A &= (p_1^A - f_1 + (\alpha_2 + \beta_2\lambda))\left(\frac{1}{2} + \frac{p_1 - p_1^A}{2(t_1 + \beta_1)}\right)
\end{aligned}$$

Derivative of π^A with respect to p_1^A at $(p_1^A = p_1)$ is given by:

$$\frac{\partial \pi^A}{\partial p_1^A}\Big|_{p_1^A = p_1} = \frac{1}{2} - \frac{(p_1 - f_1 + (\alpha_2 + \lambda\beta_2))}{2(t_1 + \beta_1)}$$

The above expression has to be non-positive for p_1 to be symmetric equilibrium price. Thus

$$p_1 \geqslant f_1 + t_1 + \beta_1 - (\alpha_2 + \lambda\beta_2)$$

For the existence of p_1 we have to put another restriction.

A6. $t_1 + \beta_1 \leqslant (\alpha_2 + \lambda\beta_2)$. This assumption insure that the expression $f_1 + t_1 + \beta_1 - (\alpha_2 + \lambda\beta_2)$ is less than f_1, so that the equilibrium value of p_1 exist.

We are now in the position to define the symmetric equilibria for configuration-1.

Proposition 3. Assume A1 to A6 hold and $\alpha_1, \alpha_2 > 0$. Then there exist a range of symmetric equilibrium (p_1, p_2) such that $p_2 = \frac{\alpha_2 + (\lambda-2)\beta_2}{2}$ and p_1 is lying in the interval given by -

$$\max\{0, f_1 + t_1 + \beta_1 - (\alpha_2 + \lambda\beta_2)\} \leqslant p_1 \leqslant f_1$$

Group-1 is single homing and group-2 is multi homing.

The profit of both the platform A and B will be equal in equilibrium and given by -

$$\pi = \pi^A = \pi^B = \frac{(p_1 - f_1)}{2} + (p_2 - f_2)$$

From proposition 3, it is clear that $(p_1 - f_1) \leqslant 0$, thus the maximum value of profit for the platform will happen when $p_1 = f_1$. And maximum profit is given by -

$$\pi^{max} = \frac{\alpha_2 + (\lambda - 2)\beta_2}{2} - f_2$$

Expression for maximum profit without congestion i.e. $(\beta_2 = 0)$ is given by-

$$\pi^o = \frac{\alpha_2}{2} - f_2$$

So the condition when congestion is beneficial (fetch more profit) for the platform is given by - $\pi^{max} - \pi^o > 0$.

Proposition 4. Suppose assumptions A1 to A6 holds and pricing at equilibrium is given by proposition 3, then the condition under which platform earn more profit because of congestion is given by -

$$\lambda > 2$$

This means, in case of competitive bottleneck with congestion, higher is tolerance parameter higher is the profit for the platforms. Since in this case, platforms do not earn any profit from group-1 and all the profits come from group-2. And the decision of group-2 agent to join any platform is independent of whether they already join any platform or not. Therefore, if tolerance of agents of group-2 is high, utility earn by group-2 agent will also be higher and the platform can extract higher profit. And when $\lambda > 2$, the effect of congestion on profit of platform get reversed.

Remark: From A3, we knew that $\lambda < 1 + \left(\frac{t_1 - \alpha_1}{\beta_1}\right)$. If $\left(\frac{t_1 - \alpha_1}{\beta_1}\right) < 1$, then $\lambda < 2$ always. It means when $t_1 < (\alpha_1 + \beta_1)$, then congestion will always reduce the profits of platform. Therefore, $t_1 > (\alpha_1 + \beta_1)$ is necessary condition for congestion to improve the profits of platform. But it is not a sufficient condition.

5 Conclusion

By incorporating congestion in the utility function of agents, cross group externalities (effective network effect) and effective transportation cost both get increased. In case of single homing, paper finds out that when tolerance parameter is less than one, overall profits of both the platforms is higher because of congestion. Unlike in single-homing case, in multi-homing there exist non-symmetric equilibrium where all the agents of one group join only one platform. But the paper finds out only symmetric equilibrium for multi-homing. Condition under which congestion is profitable for platform get reversed in multi-homing case. Now when tolerance parameter is greater than two, then profits of both the platform is increased due to congestion. This paper is a static analysis, and it is difficult to capture overall dynamics of congestion on the platform. With time, tolerance parameter of agents change, so is the overall profit of the platform. Thus one plausible extension of this work is to make tolerance parameter dynamic in nature instead of static. The end of Section 3 gives a brief interpretation of how a changing tolerance parameter affects the static model.

6 Reference

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