First-Party Content with Seller Competition in Two-Sided Markets*

Swapnil Sharma

Abstract

Two-sided platforms often provide first-party content to attract users and overcome the coordination problem created by cross-group network effects. This paper studies that decision when sellers on the platform compete with one another. Seller competition lowers each seller's profit per buyer and can lead a platform facing favorable expectations to restrict seller participation. A platform facing unfavorable expectations must instead eliminate the zero-participation equilibrium, either by subsidizing sellers or by attracting buyers directly. The comparison of first-party-content investment across these expectation regimes is no longer determined solely by whether first-party content complements or substitutes for seller participation. Under a seller-subsidy strategy, it depends on the relative marginal return to content in the two regimes. Under a buyer-attraction strategy, it also depends on how content affects the surplus extracted from sellers. The model therefore permits an unfavorable-expectations platform to invest more in first-party content even when content and seller participation are complements. A numerical example produces this outcome under a buyer-attraction strategy.

*I sincerely thank Prof. Rupayan Pal for his useful and insightful suggestions throughout the development of the project. I thank the other faculty members at IGIDR for their guidance and valuable feedback. I also thank Sumit Shrivastva for useful discussions and the participants in the IGIDR Dissertation Workshop for helpful comments.

1 Introduction

Two-sided platforms need participation on both sides of the market. Buyers are more willing to join when they expect a broad set of sellers, while sellers are more willing to enter when they expect enough buyers. These cross-group effects can support a high-participation equilibrium, but they can also leave a platform at an equilibrium with few or no users. A platform may respond by providing its own content or services, often without a separate charge, to make participation attractive before the other side has arrived. Examples include bundled games on a console, built-in applications in an operating system, product information and reviews on a marketplace, and platform-provided tools that help users complete transactions.

This paper asks how competition among sellers changes a platform's investment in such first-party content. The question matters because attracting more sellers has two effects. It gives buyers more value, but it also divides demand among a larger number of sellers and reduces each seller's profit per buyer. A platform may therefore prefer an interior level of seller participation rather than the largest possible seller base. Once seller participation becomes a strategic margin, the return to first-party content depends on more than whether that content complements or substitutes for sellers' products.

The analysis considers a monopoly platform connecting heterogeneous buyers with identical sellers. The platform chooses a buyer fee, a seller fee, and first-party content before buyers and sellers decide simultaneously whether to participate. First-party content can complement seller participation, as when platform services make third-party products easier to discover or use. It can instead substitute for seller participation, as when bundled software or games compete with third-party products. Seller competition is represented by a per-buyer profit function that declines with the number of participating sellers.

As in Hagiu and Spulber [2013], the platform may face favorable or unfavorable expectations. Under favorable expectations, buyers and sellers coordinate on the highest sustainable participation level. Under unfavorable expectations, they coordinate on the lowest level unless the platform chooses prices and content that remove the zero-participation equilibrium. The platform can do this by subsidizing sellers directly or by setting the buyer price and content level so that enough buyers join even when no seller is initially expected to participate. The paper refers to these responses as the seller-subsidy and buyer-attraction strategies.

Seller competition changes both regimes. Under the concavity condition derived in the paper, the favorable-expectations equilibrium can involve less than full seller participation. The platform

then trades off the value that additional sellers create for buyers against the reduction in profit earned by each seller. In the unfavorable-expectations regime, eliminating the zero-participation equilibrium can require the platform to attract all sellers. The unfavorable-expectations platform may therefore serve more sellers than the favorable-expectations platform, even though it begins from a more difficult coordination problem.

The main result concerns first-party-content investment across the two regimes. Without seller competition, Hagiu and Spulber [2013] show that the complementarity or substitutability between first-party content and seller participation determines the direction of the investment comparison. With seller competition and interior seller participation under favorable expectations, that relationship is no longer sufficient. Under a seller-subsidy strategy, the comparison depends on the marginal return to content in the favorable and unfavorable regimes. Under a buyer-attraction strategy, it also depends on how content changes the surplus the platform can extract from sellers. An unfavorable-expectations platform can therefore invest more in first-party content even when content and seller participation are complements. The numerical example produces exactly this outcome: the favorable platform chooses $x_f = 3$, while the unfavorable platform chooses the buyer-attraction strategy and invests $x_2 = 4$.

The paper contributes to the literature by joining two decisions that are often studied separately: using first-party content to solve a coordination problem and limiting seller participation to manage competition within the platform. The result is a more conditional account of first-party-content investment. Expectations still affect the platform's strategy, but their effect operates through the seller participation that the platform induces and the surplus available on each side.

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 presents the model. Section 4 describes simultaneous entry and the possible participation equilibria. Section 5 characterizes the platform's choices under favorable and unfavorable expectations. Section 6 provides a numerical example, and Section 7 concludes. The appendix describes additional equilibrium configurations and contains the proofs.

2 Literature Review

The coordination problem in two-sided markets provides the starting point for the analysis. Caillaud and Jullien [2003] show how an intermediary can use subsidies and non-exclusive access to overcome the difficulty of attracting two interdependent groups. Hagiu [2006] demonstrates that commitment

and the timing of participation affect platform pricing when users can coordinate on different participation outcomes. These papers establish why a platform may need to sacrifice revenue on one side to create participation on the other.

Hagiu and Spulber [2013] introduce first-party content as an additional coordination instrument. Their platform can supply content that complements or substitutes for third-party content, and they compare investment under favorable and unfavorable expectations. In their framework, sellers do not compete with one another. The nature of the interaction between first-party content and seller participation therefore gives a clear ranking of investment across expectation regimes. The present paper keeps their coordination problem but allows each seller's profit per buyer to fall as more sellers join. That change makes seller participation endogenous at an interior level and breaks the direct link between complementarity and the investment ranking.

The paper also relates to work on competition within a platform. Belleflamme and Toulemonde [2009] show that positive cross-group effects can coexist with negative effects within a group. Hagiu [2009] studies how seller competition and product variety shape platform pricing, while Belleflamme and Peitz [2019] show that a platform may restrict seller participation to protect seller margins and extract more revenue. The mechanism here is similar: entry raises buyer utility but lowers each seller's operating profit. This paper uses that tradeoff to study first-party-content investment rather than platform access alone.

Coordination bias provides another response. Hałaburda and Yehezkel [2016] show that small changes in expectations can alter a platform's preferred pricing strategy. Veiga et al. [2017] allow a platform to target users according to the cross-group benefits they create. Hagiu and Wright [2015] compare marketplace and reseller modes and link the preferred organizational form to the platform's information and control advantages. These contributions vary the platform's pricing, targeting, or business model. The present paper holds the marketplace structure fixed and asks how seller competition changes the joint choice of access prices and first-party content.

Recent work has brought seller responses directly into the analysis of first-party content. Wei et al. [2023] distinguish complementary from substitutive first-party content on a business-to-business platform and show, theoretically and empirically, that the effect of substitutive content on market concentration depends on the level of complementary content. At low levels of complementary content, substitutive content crowds out third-party products; at higher levels, sellers respond by posting more products. Their focus is market concentration and seller response. The present paper instead studies equilibrium selection and compares the platform's optimal content investment under

favorable and unfavorable expectations.

A related literature studies platforms that sell their own products alongside third-party sellers. Etro [2021] examines when a marketplace should retail first-party or private-label products rather than rely on commissions from third-party sellers. Hagiu et al. [2022] study the welfare effects of the platform's dual role and distinguish a ban on platform selling from narrower restrictions on imitation and self-preferencing. Anderson and Bedre-Defolie [2024] allow endogenous seller entry and show how a hybrid platform's own products can raise seller fees and reduce variety. These models treat first-party products as a source of sales revenue and direct product-market competition. The first-party content in this paper is chosen as a coordination device; its effect on seller competition enters through participation and seller profit per buyer.

The paper's contribution is therefore specific. It shows that once seller competition can induce an interior seller base, the complement-or-substitute classification does not by itself determine how expectations affect first-party-content investment. The relevant comparison also depends on the marginal surplus generated by content and on whether the platform removes the zero-participation equilibrium through seller subsidies or buyer attraction.

3 The Setup

Consider that there is a monopoly platform that connects buyers and sellers on each side of the platform and charges p and w to each buyer and seller, respectively. There is a continuum of buyers and sellers. Buyers denoted by i are uniformly distributed on $[0, 1]$ and sellers denoted by j is uniformly distributed over $[0, M]$.

Let us assume that buyers are heterogeneous. Buyer i obtains utility U_i by joining the platform, which is as follows

$$U_i(m, x, p) = u(m, x) - p - i;$$

where m is the number of sellers on the platform, x denotes the first-party content provided by the platform, and i denotes buyer i reservation utility. $u(m, x)$ represent the buyer's preference over m and x. We assume that $u(.,.)$ is twice continuously differentiable, increasing, concave, or linear in both arguments. A buyer joins the platform only if his utility from joining the platform is more than or equal to zero.

Sellers are assumed to be identical, and utility of each seller from joining the platform is given by

$$V(n, m, x, w) = \pi(m, x).n - w - \phi;$$

where $\pi(m, x)(> 0)$ is the profit per buyer earned by each seller, $n \in [0, 1]$ represents the number of buyers participating on the platform, and $\phi(> 0)$ is the fixed cost of each seller to port store on the platform. $\pi(.,.)$ is decreasing in m because competition among the sellers will reduce the per-buyer profit for each seller. $\pi(.,.)$ can be increasing or decreasing in x depending upon whether x and m are complement or substitute for each other.

First party content: First party content x and sellers participation m may be a complement or substitute of or independent from each other.

Definition 1

Complements: x and m are complement to each other if $\partial^2 u/\partial m \partial x > 0$ and $d\pi/dx \geq 0$

Substitutes: x and m are substitute to each other if $\partial^2 u/\partial m \partial x < 0$ and $d\pi/dx \leq 0$

Independent: x and m are independent to each other if $\partial^2 u/\partial m \partial x = 0$ and $d\pi/dx = 0$

4 Monopoly Platform with Simultaneous Entry

The platform's objective is to maximise its profit when buyers and sellers join it simultaneously. The stages of the game is as follows

- **Stage 1:** The Platform decides the amount of first party content x, price for each buyer p, and the price for each seller w; which are observable and verifiable by all.
- **Stage 2:** Each buyer and seller decide simultaneously and independently whether to join or not after observing the first party content and price vector set by platform.
- **Stage 3:** Sellers on the platform engage in oligopoly competition.

We solve the game by backward induction. Note that in the absence of seller competition (as in Hagiu and Spulber (2013)), for any given(p, w, x), number of buyer participation is given by

$$n(m) = \max\{u(m, x) - p, 0\} \tag{1}$$

And sellers participation is given by

$$m(n) = \begin{cases} 0 & n < w + \phi/\pi(x) \\ M & n \geq w + \phi/\pi(x) \end{cases} \tag{2}$$

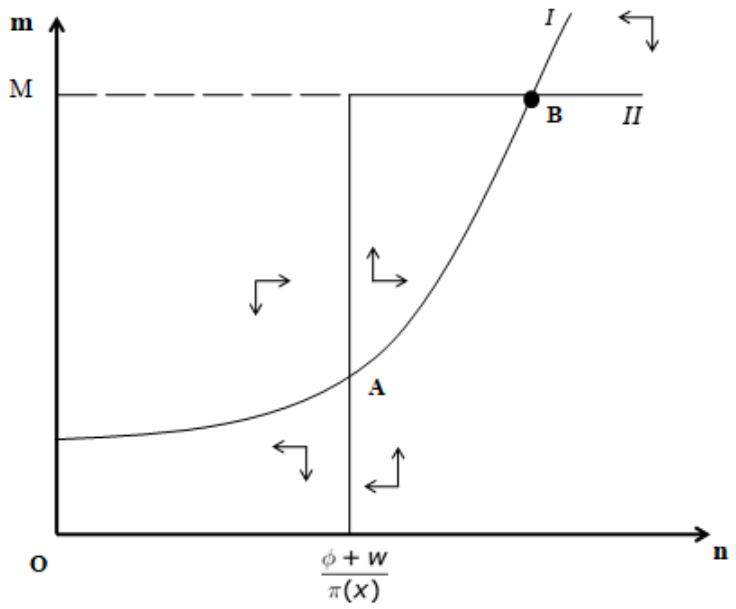


Figure 1: Participation decision of stage 2 without seller competition

In figure 1, we denote buyer participation (equation 1) by curve I and sellers participation (equation 2) by curve II. Equilibrium will occur when the expectation of buyers about the sellers participation coincides with expectation of sellers for the buyers participation. It is evident from the figure 1, that the stable equilibrium will either have zero participation of buyers and sellers (unfavourable equilibrium) or full participation of sellers (favourable equilibrium).

Now, in the presence of seller competition, the buyer participation is still given by equation 1. But the seller participation equation will change. For defining the equation of seller participation, we will first define few things

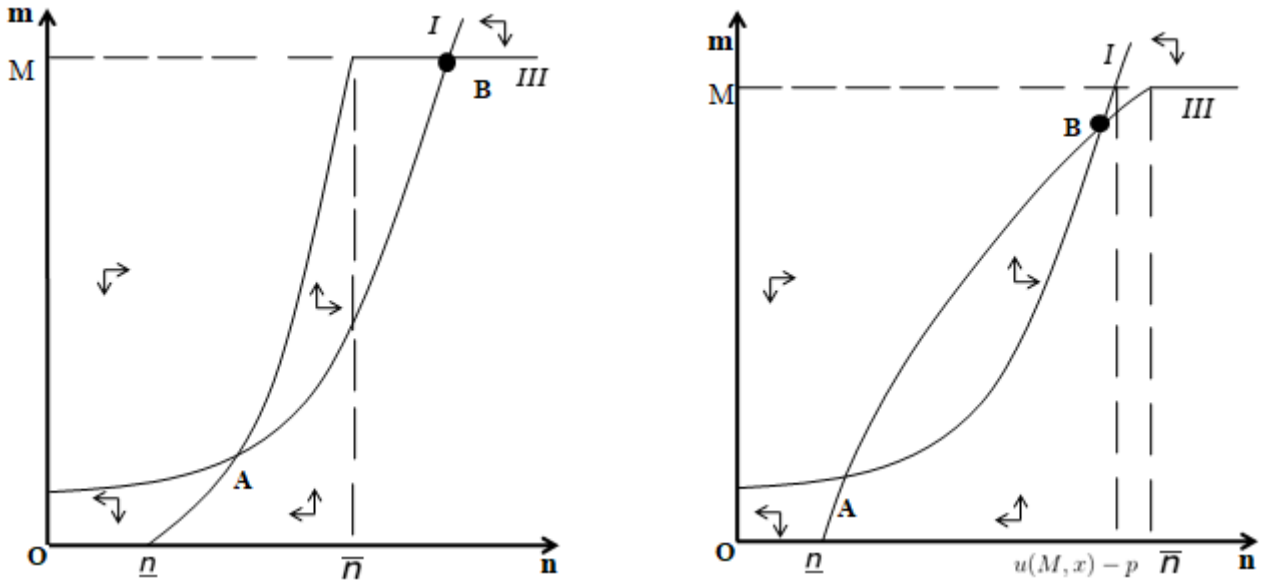
$$\underline{n} = \frac{w + \phi}{\pi(x, 0)}; \qquad \overline{n} = \frac{w + \phi}{\pi(x, M)}$$

where for any given (p, w, x), $\underline{n}$ represents the minimum number of buyers that is required on the platform to make it profitable for sellers to join the platform. Similarly, $\overline{n}$ represent the minimum number of buyers that is required for full participation of sellers on the platform. Thus when buyer participation on the platform is less than or equal to $\underline{n}$, seller participation will be zero. And when buyer participation is more than or equal to $\overline{n}$, all the sellers (M) will join the platform. For a given (p, w, x), sellers participation when the number buyer on the platform lies between $\underline{n}$ and $\overline{n}$ will be given by $V(n, m) = \pi(m, x).n - w - \phi = 0$. Thus the seller participation equation in stage 2 will be

given by

$$m(n) = \begin{cases} 0 & n \leq \underline{n} \\ \pi(m,x)n = w + \phi & \underline{n} < n < \overline{n} \\ M & n \geq \overline{n} \end{cases} \tag{3}$$

Thus, the buyer and seller participation in the presence of seller competition on the platform will be given by equation 1 and 3 respectively. Let us denote the seller participation in the presence of seller competition by curve III. Curve III can have either convex or concave shape as shown in figure 2(a) and 2(b) respectively. We can easily verify that when the shape of curve III is convex, there exist three equilibria - two stable equilibria and one unstable equilibrium. Two Stable equilibria are no participation and full participation from sellers side, denoted by O and B respectively and one unstable equilibrium will be denoted by A. But when curve III is concave, as shown in figure 2(b), the stable equilibrium (apart from no participation) can have participation lower than full participation (M) from seller side. This means that, it is in benefit of favorable platform to serve lower number of sellers. Let us denote this equilibrium as non-full equilibrium. When curve III is concave there will exist many different possibility, which we will discuss in appendix.



(a) Convex Shape of Curve III

(b) Concave Shape of Curve III

Figure 2: Participation Decision of Stage 2 With Seller Competition

5 Equilibrium Characterization with Non-Full Equilibrium

First we will derive the condition under which shape of curve III will be concave and then we will put the restriction on parameters under which it is profitable for the platform to serve lower number of sellers on the platform. The number of sellers on the platform as a function of number of buyers in second stage is given by equation 3. From equation 3 we can find the slope of curve III when $n \in (\underline{n}, \overline{n})$, which is given by

$$\left.\frac{\partial m}{\partial n}\right|_{III} = \frac{-\pi(m,x)}{n.\pi_m(m,x)}$$

where $\pi_m(m,x)$ denote the first derivative of $\pi(.,.)$ with respect to m. Since $\pi_m(m,x) < 0$, slope of curve III is positive. The shape of curve III in stage 2 will be concave only when $\partial^2 m/\partial n^2 \leq 0$.

$$\left.\frac{\partial^2 m}{\partial n^2}\right|_{III} = \frac{-1}{(n.\pi_m(m,x))^2}\{n.(\pi_m^2(m,x) - \pi(m,x)\pi_{mm}(m,x))\frac{\partial m}{\partial n} - \pi(m,x)\pi_m(m,x)\}$$

After substituting the $\partial m/\partial n$ from the above we get

$$\left.\frac{\partial^2 m}{\partial n^2}\right|_{III} = \frac{-n\pi(m,x)}{(n.\pi_m(m,x))^3}\{\pi(m,x)\pi_{mm}(m,x) - 2\pi_m^2(m,x)\}$$

Thus necessary condition for $\partial^2 m/\partial n^2 \leq 0$ is $\pi(m,x)\pi_{mm}(m,x) \leq 2\pi_m^2(m,x)$ and sufficient condition is $\pi_{mm}(m,x) \leq 0$.

Assumption 1: The profit function ($\pi(m,x)$) of each seller is such that, for any given x and m, $\pi(m,x)\pi_{mm}(m,x) \leq 2\pi_m^2(m,x)$.
Assumption 1 ensure that shape of curve III will be concave in stage 2. Stricter condition for ensuring that will be $\pi_{mm}(m,x) \leq 0$. We will called this as sufficiency condition for concavity of curve III.

Now if Assumption 1 holds, then under following conditions platform will serve lower than full participation of sellers for given (p,x,w) are -
Condition 1: $u(M,x) - p > \underline{n}$
Condition 2: $u(M,x) - p < \overline{n}$

$u(0,x) - p$ represent the number of buyers who will join the platform if none of the seller join the platform in stage 2, for any given (x,p,w). Similarly, $u(M,x) - p$ represent the number of buyers when all of the sellers join the platform. Condition 1 represent that the number of buyers when they expect all of the sellers to join the platform, should be more than minimum number of

buyers that is required to make it profitable for sellers to join the platform. Similarly, condition 2 implies that the number of buyers when they expect all of the sellers to join the platform, should be less than minimum number of buyers that is required for full participation of sellers on the platform. In the graph, conditions 1 and 2 will ensure that the curve I intersect the $m = M$ line after $\underline{n}$ and before $\overline{n}$.

5.1 Favourable expectation Equilibrium

The platform will maximise its profit with a participation constraint for the sellers. Buyers are heterogeneous and buyer participation for a given p will be either 0 or $u(m, x) - p$. But since favourable platform do not face low participation equilibria, the number of buyer on the platform will be given by $n = u(m, x) - p$. If assumption 1 holds and conditions 1 and 2 satisfy, then in the equilibrium, platform will serve less than M sellers on the platform. The objective function of the platform is given by

$$\Pi^f = p[u(m, x) - p] + w.m - c(x) \tag{4}$$

where $c(x)$ is the cost of providing first party content. It is assumed that $c(x)$ is increasing and convex in x. Platform will maximise its profit function with a constraint that it is profitable for sellers to join the platform, i.e. $\pi(m, x)n - w - \phi \geq 0$. In the equilibrium platform will set $w = \pi(m, x)n - \phi$ such that constraint will became binding. After substituting w and n, the objective function will become

$$\max_{p,x,m} \Pi^f(x, p, m) = [p + \pi(m, x).m][u(m, x) - p] - m\phi - c(x) \tag{5}$$

First order conditions will give us the equilibrium price charged to each buyer denoted by p_f; level of first party content denoted by x_f; and number of sellers served on platform denoted by m_f and are given by

$$\frac{\partial \Pi^f}{\partial p} = 0 \quad \Rightarrow \quad p_f = \frac{u(m_f, x_f) - m_f \pi(m_f, x_f)}{2}; \tag{6}$$

$$\frac{\partial \Pi^f}{\partial x} = 0 \quad \Rightarrow \quad \frac{[u(m_f, x_f) + m_f \pi(m_f, x_f)][u_x(m_f, x_f) + m_f \pi_x(m_f, x_f)]}{2} = c'(x_f); \tag{7}$$

$$\frac{\partial \Pi^f}{\partial m} = 0 \quad \Rightarrow \quad \frac{[u(m_f, x_f) + m_f \pi(m_f, x_f)][\pi(m_f, x_f) + u_m(m_f, x_f) + m_f \pi_m(m_f, x_f)]}{2} = \phi \tag{8}$$

Before discussing these three first order conditions, we will define a term which will help in understanding these equations. Let us define a variable $G(m, x)$, given by

$$G(m, x) = u(m, x) + m\pi(m, x) \tag{9}$$

$G(m, x)$ can be interpreted as per buyer gross surplus generated on the platform by the participation of m sellers and x amount of first party content.

$$G_x(m, x) = u_x(m, x) + m\pi_x(m, x); \qquad G_m(m, x) = u_m(m, x) + \pi(m, x) + m\pi_m(m, x) \tag{10}$$

Since $c'(x)$ and ϕ are positive, we can easily verify from equation 7 and 8 that $G_x(m_f, x_f)$ and $G_m(m_f, x_f)$ will also be positive (as $[u(m, x) + m\pi(m, x)] > 0$ for every x and m). Since $w = p[u(m, x) - p] - \phi$, therefore equilibrium price charged to each seller, denoted by w_f, will be given by

$$\begin{aligned} w_f &= -\frac{[u(m_f, x_f) + m_f \pi(m_f, x_f)][u_m(m_f, x_f) + m_f \pi_m(m_f, x_f)]}{2} \\ \Rightarrow w_f &= \frac{[u(m_f, x_f) + m_f \pi(m_f, x_f)][\pi(m_f, x_f) - G_m(m_f, x_f)]}{2} \end{aligned} \tag{11}$$

Thus, the equilibrium price for each seller,w_f, given by equation 11, could be negative even in case of favourable equilibrium, when $G_m(m_f, x_f) > \pi(x_f, m_f)$.

Lemma 1: If assumption 1 and condition 1 and 2 holds, then the optimal first party content,x_f and seller's participation, m_f for the favourable platform will be given by equation 7 and 8 respectively. If for a given x_f and m_f, $G_m(m_f, x_f) > \pi(x_f, m_f)$ then favourable platform will subsidize the joining of sellers on the platform.

Lemma 1 implies that when per buyer gross surplus generated on the platform increases with sellers with a rate more than per buyer profit for each seller, the platform will subsidies the joining of sellers and extract the gross surplus from buyers. This is in alignment with the result of Armstrong [2006], Rochet and Tirole [2006] where they have predicted that the group which is tougher

to attract will be subsidized or charged lower. Here, because of competition among sellers, per buyer profit for each seller decreases. Therefore, it is tougher to attract further sellers on the platform after a certain number of sellers and the condition under which it will happen, will be given by $\pi(m,x) < G_m(m,x)$ for a given x. Although, this result is similar to earlier established results, the novelty of this result here is, the condition under which favourable platform subsidies the joining of sellers is derived endogenously in the model.

Note 1: Even when joining of sellers is subsidized, $w_f + \phi > 0$[1]. Thus the $\underline{n}$ and $\overline{n}$ will be positive. Importance of note 1 will be discussed later under unfavorable expectation case. Conditions 1 and 2 should be satisfy at equilibrium value of (x_f, p_f, w_f). Form equation 6, we have p_f in terms of x_f and m_f, thus we can express conditions 1 and 2 in terms of x_f and m_f solely. Let us denote the favourable equilibrium value of $\underline{n}$ and $\overline{n}$ by $\underline{n}_f$ and $\overline{n}_f$ respectively. Equilibrium value of $\underline{n}_f$ and $\overline{n}_f$ will be given by

$$\overline{n}_f = \frac{w_f + \phi}{\pi(M, x_f)} = \left[\frac{u(m_f, x_f) + m_f \pi(m_f, x_f)}{2}\right] \frac{\pi(m_f, x_f)}{\pi(M, x_f)}$$

$$\underline{n}_f = \frac{w_f + \phi}{\pi(0, x_f)} = \left[\frac{u(m_f, x_f) + m_f \pi(m_f, x_f)}{2}\right] \frac{\pi(m_f, x_f)}{\pi(0, x_f)}$$

Now, we have to check that there must exist some pair of (m_f, x_f) for which the conditions 1 and 2 will be satisfy.

Condition 1: $u(M, x_f) - p_f > \underline{n}_f$ will reduce to

$$2u(M, x_f)\pi(0, x_f) + m_f \pi(m_f, x_f)[\pi(0, x_f) - \pi(m_f, x_f)] > u(m_f, x_f)[\pi(0, x_f) + \pi(m_f, x_f)]$$

Since $\pi(0, x_f) > \pi(m_f, x_f)$ and $u(M, x_f) > u(m_f, x_f)$ for all $m_f < M$, this condition will always be true for any pair of (m_f, x_f).

Condition 2: $u(M, x_f) - p_f < \overline{n}_f$ will reduce to

$$2u(M, x_f)\pi(M, x_f) + m_f \pi(m_f, x_f)[\pi(M, x_f) - \pi(m_f, x_f)] < u(m_f, x_f)[\pi(M, x_f) + \pi(m_f, x_f)] \tag{12}$$

Since $\pi(m_f, x_f) > \pi(M, x_f)$ and $u(M, x_f) > u(m_f, x_f)$, for all $m_f < M$ the above expression will be hold for some pair of (m_f, x_f). Equilibrium (m_f, x_f) must satisfy the equation 12.

[1] $w_f + \phi = [u(m_f, x_f) + m_f \pi(m_f, x_f)]\pi(m_f, x_f)/2$. Since $u(m, x)$ and $\pi(m, x)$ are positive for any given m and x, thus $w_f + \phi > 0$.

5.2 Unfavorable Expectation Equilibrium

When platform faces an unfavourable expectation, to make positive profit, platform will set the price charged by each buyer and seller and invest in first party content in such a way that it eliminate the low (or zero) participation equilibrium. Therefore, for seller participation, the platform will set the price for sellers such that it is profitable for each seller to join the platform even if they expect that none of other sellers will join the platform. That is, for any given first party content x, price charged to each seller, w will be

$$\pi(0, x) \max[u(0, x) - p, 0] - w - \phi \geq 0$$

Let us denote the equilibrium value of first party content and price charged to buyer in unfavourable equilibrium by x_{uf} and p_{uf} respectively. Depending upon $u(0, x_{uf})$ and p_{uf}, three cases can arise. These three cases are - either $u(0, x_{uf}) < p_{uf}$ or $u(0, x_{uf}) = p_{uf}$ or $u(0, x_{uf}) > p_{uf}$. It is easy to show that for any optimal x_{uf} and p_{uf}, it is not possible that $u(0, x_{uf}) = p_{uf}$. This is because if $u(0, x_{uf}) = p_{uf}$, then it will require $u(M, x_{uf}) \leq 2u(0, x_{uf})$ and $u(m_{uf}, x_{uf}) \geq 2u(0, x_{uf}) + m_{uf}\pi(0, x_{uf})$ where m_{uf} is equilibrium of sellers on the platform in case of unfavourable equilibrium which can be less than or equal to M. Since $u(m, x)$ is strictly increasing in m, therefore both cannot happen simultaneously. Thus only two case will arise. We will call the case $u(0, x_{uf}) < p_{uf}$ as seller subsidy strategy and $u(0, x_{uf}) > p_{uf}$ as buyer attraction strategy. We will see below why it is called so.

Case 1: Seller Subsidy Strategy

In this case $u(0, x_{uf}) < p_{uf}$, thus price charged to each seller for non zero participation will be less than or equal to $-\phi$. Let us denote the equilibrium price charged to each buyer and seller and investment in first party content for seller subsidy case by p_1, w_1, and x_1 respectively. The price charged to each seller should be $w_1 \leq -\phi$, for avoiding zero participation. In the equilibrium, the constraint will be binding and the price charged to each seller will be $w_1 = -\phi$. Since ϕ is positive number, so platform will always subsidies the joining of seller for avoiding no participation equilibrium. That is why it is known as seller subsidy strategy. Utility of each seller when m number of sellers join the platform is given by

$$v(x, p, w, m) = \pi(m, x)n - w - \phi$$

Since $w = -\phi$, $v(x, p, w, m) \geq 0$ for any given number of buyers or sellers on the platform. Thus it is beneficial for each sellers to join the platform. Therefore, number of sellers on the platform in seller

subsidy strategy will be always M. Here, $\underline{n}$ and $\overline{n}$ reduced to zero (in favourable expectation which is always greater than zero, note 1). Now the objective function for the platform facing unfavourable expectation with seller subsidy will be

$$\max_{p,x} \Pi^1 = p[u(M,x) - p] - M\phi - c(x)$$

The equilibrium buyer's price p_1 and first party content x_1 can be obtain form first order condition and are given by

$$p_1 = \frac{u(M,x_1)}{2}; \text{and} \qquad \frac{[u(M,x_1)u_x(M,x_1)]}{2} = c'(x_1) \tag{13}$$

In equilibrium, the first party content and buyer price must satisfy the condition $u(0,x_{uf}) < p_{uf}$ i.e. $p_1 > 2u(0,x_1)$. On comparing the (p_f, w_f) with (p_1, w_1), we easily conclude following things

- Since $w_f > -\phi$ (note 1), thus $w_f > w_1$. That is, price charged to the sellers when platform is facing favourable expectation is always more than when the platform facing unfavourable expectation and adopt seller subsidy strategy.
- For any given first party content x, $p_1 > p_f$. That is, for any given first party content, buyers are charged more in case when platform faced unfavourable expectation and adopt seller subsidy strategy in comparison to favourable expectation.

From above two points we can say that when platform face unfavourable expectation and it is difficult to attract sellers on the platform, platform charge lower (or subsidies more) for the joining of sellers and extract some part of loss by charging higher fee from the buyers.

Case 2: Buyer attraction Strategy

In this case $u(0,x_{uf}) > p_{uf}$, now the price charged to the each seller will be -

$$w = \pi(0,x)[u(0,x) - p] - \phi$$

Let us denote the equilibrium price charged to each buyer and seller and investment in first party content for buyer attraction strategy will be given by p_2, w_2, and x_2 respectively. In buyer attraction strategy case, platform can serve M or less than M number of sellers. Condition for full participation of sellers will be $u(M,x_2) - p_2 \geq \overline{n}$. Now with the assumption that $u(0,x_2) > p_2$ and $u(M,x_2) - p_2 \geq \overline{n}$ in the equilibrium, the objective function of unfavourable platform with buyer attraction strategy will be -

$$\max_{p,x} \Pi^2 = p[u(M,x) - p] + M[\pi(0,x)(u(0,x) - p) - \phi] - c(x)$$

The equilibrium buyer's price, seller's and first party content can be obtain from first order conditions and will be given by

$$\begin{aligned} p_2 =& \frac{u(M,x_2) - M\pi(0,x_2)}{2}; \\ w_2 =& \pi(0,x_2)\left[\frac{2u(0,x_2) - u(M,x_2) + M\pi(0,x_2)}{2}\right] - \phi; \\ c'(x_2) =& \frac{(u(M,x_2) - M\pi(0,x_2))(u_x(M,x_2) - M\pi_x(0,x_2))}{2} + \\ & M[\pi_x(0,x_2)u(0,x_2) + \pi(0,x_2)u_x(0,x_2)] \end{aligned} \tag{14}$$

Now, $u(0,x_2) > p_2$ will reduce to $u(M,x_2) < 2u(0,x_2) + M\pi(0,x_2)$ and $u(M,x_2) - p_2 \geq \overline{n}$ will become

$$u(M,x_2) \geq 2\left[\frac{u(0,x_2)\pi(0,x_2) - u(M,x_2)\pi(M,x_2)}{\pi(0,x_2) - \pi(M,x_2)}\right] + M\pi(0,x_2) \tag{15}$$

Thus when $u(M,x_2) < 2u(0,x_2) + M\pi(0,x_2)$, buyer attraction strategy is feasible for platform facing unfavourable expectation and when equation 16 satisfy, the platform will serve M sellers (or full sellers participation) on the platform in buyer attraction strategy.

Few points of interest here. First, since $u(M,x_2) < 2u(0,x_2) + M\pi(0,x_2)$ so from the equation of w_2 we can see $w_2 + \phi > 0$. Thus $\underline{n}$ and $\overline{n}$ both will be positive and $w_2 > w_1$. Second, $p_2 < p_1$ for any given value of first party content. However we can not exactly establish the similar relation between p_f and p_2 because of presence of seller competition. Third $\left[\frac{u(0,x_2)\pi(0,x_2) - u(M,x_2)\pi(M,x_2)}{\pi(0,x_2) - \pi(M,x_2)}\right] < u(0,x_2)$, thus we say whenever

$$\left[\frac{u(0,x_2)\pi(0,x_2) - u(M,x_2)\pi(M,x_2)}{\pi(0,x_2) - \pi(M,x_2)}\right] + M\pi(0,x_2) \leq u(M,x_2) < 2u(0,x_2) + M\pi(0,x_2)$$

buyer attraction strategy is feasible and platform will serve all the seller. In buyer attraction strategy, platform will set price (and first party content) for buyer in such a way that even in absence of any seller there should be enough buyer demand. This will make attraction of sellers on the platform easier. Thus, both the strategies (seller subsidy and buyer attraction) in unfavourable expectation in the end rely on persuading the sellers to join the platform.

When platform faces unfavourable expectation, platform adopt between two possible strategies (either any one or both strategies are feasible depending upon parameters) which is characterize by skewed pricing mechanism. Seller subsidy leads to low pricing to sellers and high price to buyer and vice verse in case of buyer attraction strategies. This is similar to Hagiu and spulber (2013), but one

thing which is very interesting to note in presence of seller competition is that - the platform facing unfavourable expectation will serve more number of sellers and buyers in comparison to favourable case.

Depending upon the values of parameters in unfavourable case, either seller subsidy or buyer attraction strategy or both are feasible. When both strategies are feasible, platform will choose which fetch higher profit. Following assumption lemma will characterize the platform decision of choosing which strategy to take.

Assumption 2:

$$u(M,x) \geq 2\left[\frac{u(0,x)\pi(0,x) - u(M,x)\pi(M,x)}{\pi(0,x) - \pi(M,x)}\right] + M\pi(0,x)$$

This assumption will make sure that in buyer attraction strategy all the sellers participate on the platform.

Lemma: If assumption 2 holds, then the simultaneous entry of buyers and sellers in case of unfavourable expectation will reduce to maximization of following objective function -

$$\Pi^{uf}(x) = \begin{cases} \frac{(u(M,x))^2}{4} - c(x) - M\phi & \text{if } u(M,x) \geq 2u(0,x) + \frac{M\pi(0,x)}{2} \\ \frac{(u(M,x) - M\pi(0,x))^2}{4} + M\pi(0,x)u(0,x) - c(x) - M\phi & \text{if } u(M,x) \leq 2u(0,x) + \frac{M\pi(0,x)}{2} \end{cases} \tag{16}$$

The resulting solution will be either x_1 if first condition is satisfied (seller subsidy strategy solution) or x_2, solution of buyer attraction strategy. Derivation of this is present in appendix A2.

Condition in above expression is saying that if per buyer maximum profit ($\pi(0,x)$) for all sellers is less than the change in buyer's utility when all sellers are or none of them on the platform , then it will be better for platform to adopt seller subsidy otherwise buyer attraction strategy. This is in line with Armstrong [2006], Rochet and Tirole [2006], according to which the group which is tougher to attract will be subsidies or charged lower. Here if first condition satisfy, then between buyers and sellers, it is sellers group which tougher to bring on the platform. That's why platform will adopt seller subsidy strategy. The novelty of same result in this paper is that the group which is tougher to bring on the platform is determine endogenously in presence of seller competition.

Now we are ready to discuss our main result of the paper, which determine the amount of first party content in unfavourable expectation situation with respect to favourable expectation. In Hagiu and Spulber [2013], they have derive that in the absence of sellers competition the relationship between x_f and x_{uf} will depend on the nature of x and m. That is, if x and m are complementary,

then the platform facing unfavourable expectation will invest less in x in comparison to favourable expectation case. Whereas when x and m are substitute results got reserve. However, this paper concludes that, this clear demarcation based on x and m are substitute or complement gets deplete in presence of sellers competition. Investment in x will depend on per buyer gross surplus generated and its rate of change with respect to x in favourable expectation vis-a-vis buyer utility and its rate of change with respect to first party content in case of unfavourable expectation. In mathematical term, investment in x will depend on whether $G(m_f, x)G_x(m_f, x)$ is greater than or lower than $u(M, x)u_x(M, x)$.

PROPOSITION: If unfavourable platform adopt seller subsidy strategy, it will invest less in first party content if $G(m_f, x)G_x(m_f, x) - u(M, x)u_x(M, x)$ is positive ($x_1 < x_f$) otherwise invest higher ($x_1 > x_f$) regardless of x and m are complementary or substitute. Whereas in buyer attraction, investment decision in first party content could be high or low in comparison to x_f, but when the interaction of x and m is not very strong for the buyers, then of being x and m complementary (substitute) will lead to higher (lower) investment in first party content.

Table 1: UF Platform Strategies for First Party Content

UF Platform Strategy	$G()G_x() - u()u_x()$is positive	$G()G_x() - u()u_x()$ is negative
Seller Subsidy	Lower Investment	Higher Investment
Buyer Attraction	Lower/Higher Investment	Lower/Higher Investment

Table 1 summaries the results. Derivation of results is given in appendix A3. The above results is contrary to existing result derive in absence of sellers competition especially for the case of buyer attraction strategy. In buyer attraction strategy, when interaction between x and m is not very strong for buyers i.e rate of change of buyer's utility with respect to x do not change very much with number of sellers ($\frac{\partial^2 u}{\partial m \partial x}$ is very small), then with x and m being complementary leads to higher investment in first party content. The result can be explain as follows -

Let us denote $\Delta(x)$ as difference in profit of favourable and unfavourable platform i.e. $\Delta(x) = \Pi^f(x) - \Pi^{uf}(x)$, where $\Pi^{uf}(x)$ is given by equation 3. Now unfavourable platform will choose higher investment as compare to favourable platform if this difference is decreasing in x i.e when $\Delta'(x) < 0$. In seller subsidy case, unfavourable platform does fetch any profit from the seller side, so all the profit comes from buyers. Thus $\Delta'(x)$ will be difference of rate of change of profit of favourable and surplus from buyers for unfavourable platform. That is, in seller subsidy case $\Delta'(x) =$

$G(m_f, x)G_x(m_f, x) - u(M, x)u_x(M, x)$. In absence of seller competition, favourable platform will serve all the sellers and $\Delta'(x)$ will be positive or $x_u f < x_f$ whenever x and m are complementary. But now with the seller competition and non-full participation of seller on favourable platform, $\Delta'(x)$ could be negative even when x and m are complementary. At that the same time, $\Delta'(x)$ could be positive for x and m are substitutes. Therefore in seller subsidy case, decision of first party content for unfavourable platform with respect to favourable platform will depend only on the sign of $G(m_f, x)G_x(m_f, x) - u(M, x)u_x(M, x)$ and not solely on the relation between x and m. Relation between x and m will affect the $G(m_f, x)G_x(m_f, x)$, like if $G(m_f, x)G_x(m_f, x) > u(M, x)u_x(M, x)$ when x and m are substitutes then $G(m_f, x)G_x(m_f, x)$ will definitely be greater than $u(M, x)u_x(M, x)$ when x and m are complement. But not other way around.

In case of buyer attraction strategy, unfavourable platform derive surplus from seller and could be from buyers as p_2 can be positive also. So in this case $G(m_f, x)G_x(m_f, x) - u(M, x)u_x(M, x)$ will only capture rate of change of profit for favourable platform and surplus derive from buyers of unfavourable platform. But the final decision will also depend upon how surplus from the sellers is changing with respect to x. $\Delta'(x)$ for buyer attraction is given by -

$$\Delta_2'(x) = \frac{1}{2}\Big[[G(m, x)G_x(m, x) - u(M, x)u_x(M, x)] + M\pi_x(0, x)[u(M, x) - 2u(0, x) - M\pi(0, x)] + M\pi(0, x)[u_x(M, x) - 2u_x(0, x)]\Big]$$

We know that buyer attraction is feasible when $u(M, x) < 2u(0, x) + \frac{M\pi(0,x)}{2}$, thus $u(M, x) - 2u(0, x) - M\pi(0, x)$ will be negative. Thus $\pi_x(0, x)[u(M, x) - 2u(0, x) - M\pi(0, x)]$ is negative (positive) when x and m are complements (substitutes). Also, when x and m are complements (substitutes) then $u_x(M, x) - u_x(0, x)$ is positive (negative). But we do not know the sign of $u_x(M, x) - 2u_x(0, x)$ for x and m being complements (for substitutes it will definitely be negative). So if buyer responsiveness for party content with increase in number of sellers is small i.e. $u_x(M, x) - u_x(0, x)$ is small number (positive or negative) then $u_x(M, x) - 2u_x(0, x)$ will be negative even for x and m being complementary. Thus for a given sign of $G(m, x)G_x(m, x) - u(M, x)u_x(M, x)$, when responsive of buyer's utility with respect to x with increase in m is small, then of being x and m complementary will leads to higher investment in first party content. This is quiet contrary to earlier established result.

Unfavourable platform whether follows seller subsidy or buyer attraction strategy, value and sign of $G(m, x)G_x(m, x) - u(M, x)u_x(M, x)$ played an important role in deciding first party content relative to x_f. This term will reduce to $Mu(M, x)\pi_x(M, x) + M\pi(M, x)(u_x(M, x) + M\pi_x(M, x))$

when favourable platform start serving all the sellers in the equilibrium, which is always positive if x and m are complementary. The serving of more number of sellers by unfavourable platform will leads to higher utility derive by each buyer on the unfavourable platform. And this change the factor which determine investment for unfavourable platform relative to favourable platform. The favourable platform will serve non-full number of sellers only when shape of sellers participation curve in stage 2 is concave (assumption 1) and conditions 1 and 2 hold. Thus assumption 1 and conditions 1 and 2 determine the environment in which result of Hagiu and Spulber (2013) does not holds. In presence of sellers competition with assumption 1, if condition 1 and 2 do not holds then favourable platform will serve all the sellers and we will get the result of Hagiu and Spulber (2013) for seller subsidy case. Whereas, in buyer attraction strategy, even if favourable platform serve full sellers on the platform, that is conditions 1 and 2 do not hold, if relation between seller participation and first party content is weak for buyer then complement nature will lead to higher investment. In buyer attraction strategy, effect of sellers competition on the platform is more profound than seller subsidy case. This is because, in case of buyer attraction strategy, platform extract surplus from seller side also which became difficult in presence of sellers competition.

The role of assumption 2 is to ensure full participation in buyer attraction strategy. The above result will holds even if relax the assumption 2. This is because, assumption 2 do not have any affect in case of seller subsidy strategy, so the first of result table will be same. In buyer attraction strategy, relaxation of assumption 2 will leads to lower of sellers on the platform. In that case, in the expression of $\Delta_2'(x)$, M will be replaced by m_2. But still the analysis will remain the same. Assumption 2 make the cutoff of buyer attraction and seller subsidy strategy at a fix point and expression for comparison between x_f and x_2 neat and clear.

Although this paper has not derive the expression for sequential entry of sellers and buyers, but we can easily see that the result hold true for seller subsidy strategy case in sequential entry. This is because we know from Hagiu and Spulber (2013) that, sequential entry with commitment will leads to the result of simultaneous entry. Also from the same paper we know that, for favourable platform and unfavourable platform with seller subsidy strategy, it is optimal to commit in sequential entry. Thus, result of sequential entry of sellers and buyer will be same as simultaneous entry for the seller subsidy case.

6 An Example

Let us take an example where $u(m,x) = u_o + msx; \quad \pi(m,x) = \pi_o/(m+1); \quad c(x) = c_o(x^2/2)$. In this example, $u_o > 0$ is constant which can be interpreted as intrinsic utility derived by each buyer by joining the platform; s is positive constant which represent additional utility apart intrinsic utility derive by each buyer when m and x is equal to one; $\pi_o > 0$ is also a constant which can be interpreted as maximum seller surplus which will divided among all the sellers on the platform. Here we assume $2c_o > M^2s^2$ and $u_o > M\pi_o$ for existence of equilibrium. Since

$$\frac{\partial^2 u}{\partial m \partial x} = s > 0; \quad \text{and} \quad \frac{\partial \pi}{\partial x} = 0$$

thus x and m are complement to each other. According to Haigu and Spulber (2013), in the absence of sellers competition, when x and m are complement to each other, the unfavorable platform will always invest lower in first party content regardless of whether unfavourable platform follow seller subsidy or buyer attraction strategy. But with seller competition and non-full equilibrium this will not always be the case.

From equation 7 and 8, the number of sellers and first party content will be given by

$$2\phi = (u_o + m_f x_f s + \frac{m_f \pi_o}{m_f + 1})(\frac{\pi_o}{(m_f+1)^2} + s x_f) \tag{17}$$

$$x_f = \frac{m_f s}{m_f + 1}\left(\frac{u_o + m_f(u_o + \pi_o)}{2c - m_f^2 s^2}\right) \tag{18}$$

Equation 17 and 18 simultaneously determine the value of m_f and x_f. m_f will be less than or equal to M. If equation 12 satisfy, the in equilibrium m_f will be strictly less than M. Equation 12 will be given by

$$2(m_f+1)(u_o + Msx_f) - \frac{m_f(M - m_f)}{m_f + 1} < (u_o + m_f s x_f)(M + m_f + 2) \tag{19}$$

We can easily cross verify that whenever $m_f < M$, above equation will go through. Now, the solution unfavourable platform can be obtain from equation 13 and 14 which will be given by

$$x_1 = \frac{u_o M s}{2c_0 - M^2 s^2} \quad \text{if} \quad c_o \leq \frac{2u_o M^2 s^2}{2u_o + M\pi_o} \tag{20}$$

$$x_2 = \frac{Ms(u_o - M\pi_o)}{2c_0 - M^2 s^2} \quad \text{if} \quad c_o \geq \frac{2u_o M^2 s^2}{2u_o + M\pi_o} \tag{21}$$

On comparing we comparing 17 and 20, we can easily see that $x_1 > x_f$ for some value of m_f. Similarly for x_2 by comparing 17 and 21. Let us take some numerical value to show that

Let $u_o = 100$, $c_o = 100$, $\phi = 190$ $M = 10$, $\pi_o = 6$ and $s = 1$

On solving equation 17 and 18 simultaneously with these value we get

$$x_1 = 3; \quad \text{and} \quad m_f = 5$$

We can easily check that equation 19 is satisfied with these value of x_f and m_f.

$$\frac{2u_o M^2 s^2}{2u_o + M\pi_o} = \frac{1000}{13} < c_o$$

Thus the unfavourable platform will adopt buyer attraction strategy and equilibrium first party content will be given by x_2.

$$x_2 = \frac{Ms(u_o - M\pi_o)}{2c_0 - M^2 s^2} = 4$$

Thus $x_2 = 4 > x_f = 3$, which is opposite to predicted by Hagiu and Spulber (2013).

7 Conclusion

In this paper, we have seen that the favourable platform in the presence of sellers competition, can serve lower than full participation of sellers on the platform under certain assumption and conditions. In the presence of sellers competition and non-full participation of sellers in favourable platform, the results of Hagiu and Spulber [2013] do not hold. Investment decision of unfavourable platform vis-a-vis favourable platform does not solely depends on the nature of relation between first party content and sellers participation. Investment decision in seller subsidy case with respect to favourable platform, will depend on difference between per buyer gross surplus generated multiplied by its rate of change with respect to first party content in favourable case and buyer utility multiplied by its rate of change with respect to first party content in unfavourable case. If above expression is positive, then unfavourable platform will invest lower otherwise higher in comparison to favourable case. In buyer attraction strategy, for a given sign of above expression if relation between first party content and seller participation is weak for buyers, then complement nature of first party content and sellers participation will leads to higher investment. This is quiet contrary to the result established in Hagiu and Spulber [2013].

We can obtained the result of Hagiu and Spulber [2013] for seller subsidy case even in presence of sellers competition, if we relax the conditions 1 and 2 for favourable case equilibrium. This is because, in the seller subsidy case, platform has to subsidies the joining of seller participation and subsidy make it profitable for each seller to join the platform. Now, if favourable platform serve all

the sellers, that is condition 1 and 2 do not hold, then the investment decision will depend only on the nature of first party content and sellers participation. Whereas, in buyer attraction strategy, even if favourable platform serve full sellers on the platform, if relation between seller participation and first party content is weak for buyer then complement nature will lead to higher investment. In buyer attraction strategy, effect of sellers competition on the platform is more profound than seller subsidy case. This is because, in case of buyer attraction strategy, platform extract surplus from seller side also which became difficult in presence of sellers competition.

Although this paper only deal with simultaneously entry of buyers and sellers, but we have argued that some part of the result will hold true even in case sequential entry. However, it will be interesting to see whether the result of Hagiu and Spulber [2013] for sequential entry of platform will remain valid in case of seller competition or not. This could be future line of research.

References

Simon P. Anderson and Özlem Bedre-Defolie. Hybrid platform model: Monopolistic competition and a dominant firm. *The RAND Journal of Economics*, 55(4):684–718, 2024. doi: 10.1111/1756-2171.12478.

Mark Armstrong. Competition in two-sided markets. *The RAND journal of economics*, 37(3): 668–691, 2006.

Paul Belleflamme and Martin Peitz. Managing competition on a two-sided platform. *Journal of Economics & Management Strategy*, 28(1):5–22, 2019.

Paul Belleflamme and Eric Toulemonde. Negative intra-group externalities in two-sided markets. *International Economic Review*, 50(1):245–272, 2009.

Bernard Caillaud and Bruno Jullien. Chicken & egg: Competition among intermediation service providers. *RAND journal of Economics*, pages 309–328, 2003.

Federico Etro. Product selection in online marketplaces. *Journal of Economics & Management Strategy*, 30(3):614–637, 2021. doi: 10.1111/jems.12428.

Andrei Hagiu. Pricing and commitment by two-sided platforms. *The RAND Journal of Economics*, 37(3):720–737, 2006.

Andrei Hagiu. Two-sided platforms: Product variety and pricing structures. *Journal of Economics & Management Strategy*, 18(4):1011–1043, 2009.

Andrei Hagiu and Daniel Spulber. First-party content and coordination in two-sided markets. *Management Science*, 59(4):933–949, 2013.

Andrei Hagiu and Julian Wright. Marketplace or reseller? *Management Science*, 61(1):184–203, 2015.

Andrei Hagiu, Tat-How Teh, and Julian Wright. Should platforms be allowed to sell on their own marketplaces? *The RAND Journal of Economics*, 53(2):297–327, 2022. doi: 10.1111/1756-2171.12408.

Hanna Hałaburda and Yaron Yehezkel. The role of coordination bias in platform competition. *Journal of Economics & Management Strategy*, 25(2):274–312, 2016.

Jean-Charles Rochet and Jean Tirole. Two-sided markets: a progress report. *The RAND journal of economics*, 37(3):645–667, 2006.

André Veiga, E Glen Weyl, and Alexander White. Multidimensional platform design. *American Economic Review*, 107(5):191–195, 2017.

Jianjiong Wei, Xueqi Wei, Dawei Zhang, and Jie Liu. First-party content and market concentration on a B2B e-commerce platform: Theory and empirical evidence. *Decision Support Systems*, 170: 113958, 2023. doi: 10.1016/j.dss.2023.113958.

8 Appendix

8.1 Different Equilibrium When Curve III is concave

Equilibrium will occur when curve I and curve III intersection which are given by equation 1 and 3 respectively. Buyers participation equation will be given by $n = u(m,x) - p$ and seller participation when $n \in (\underline{n}, \overline{n})$ will be given by $\pi(m,x)n - w - \phi = 0$. Let us define a function for given (x, p, w)

$$f(m) = \pi(m,x)\{u(m,x) - p\} - w - \phi$$

$f(m) = 0$ will decide the number of intersection of curve I and III when $n \in (\underline{n}, \overline{n})$. We will always get a stable equilibrium for $n \leq \underline{n}$, which is no participation equilibrium. Also, whenever $u(M,x) - p \geq \overline{n}$, we will get a stable equilibrium of full participation. This is due to fact that curve III will become horizontal for $n \geq \overline{n}$. Since curve I is always increasing and when $u(M,x) - p \geq \overline{n}$, it means that the curve I will intersect with horizontal part of curve III. Thus, with seller competition and concave shape of curve III, we will have many different sets of solution as follows

1. Only One Stable Equilibrium : No Participation Equilibrium

This will occur if one or both of the below case holds -

Case 1: $f(0) > 0$ and $f'(m) > 0 \ \forall m \in [0, M]$

Case 2: $f(0) < 0$ and $f'(m) < 0 \ \forall m \in [0, M]$

$$f'(m) = \pi_m(m,x)(u(m,x) - p) + \pi(m,x)u_m(m,x)$$
$$f''(m) = \pi_{mm}(m,x)(u(m,x) - p) + 2\pi_m(m,x)u_m(m,x) + \pi(m,x)u_{mm}(m,x)$$

Since $\pi_m(m,x)u_m(m,x) < 0$, $u_{mm}(m,x) \leq 0$ and if sufficiency condition ($\pi_{mm}(m,x) \leq 0$) holds then $f''(m)$ will negative be everywhere. Thus, if $f'(0) < 0$ then $f'(m) < 0 \ \forall m \in [0, M]$. Therefore, case 2 will give us only one stable equilibrium solution. (With case 1, it is possible to get the condition for no solution when $n \in [\underline{n}, \overline{n}]$, but it will have definitely have one more stable equilibrium when $n > \overline{n}$ which we see later).

$$f(0) < 0 \quad \Rightarrow \quad u(0,x) - p < \tfrac{w+\phi}{\pi(0,x)} \quad \text{or} \quad u(0,x) - p < \underline{n}$$

$$f'(0) < 0 \quad \Rightarrow \quad u_m(0,x) < -\pi_m(0,x)(u(0,x) - p)/\pi(0,x)$$

Last condition implies that slope of curve I should always be greater than the slope of curve II. Shape of curve I and curve III will be as shown below in figure 3.

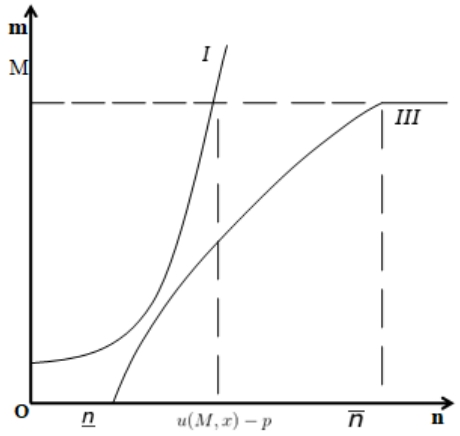


Figure 3: Only One Stable Equilibrium Case

Thus the conditions for only one stable equilibrium will be

1. $\pi_{mm}(m,x) \leq 0 \quad \forall \quad m \in [0,M]$

2.$u(0,x) - p < \underline{n}$

3.$u_m(0,x) < \frac{-\pi_m(0,x)(u(0,x)-p)}{\pi(0,x)}$

2a. Only Two Stable Equilibrium: Zero and Full Participation

Two stable equilibrium and no unstable equilibrium will exist only when there is no intersection of curve I and III when $n \in (\underline{n}, \overline{n})$. This will occur when $f(0) > 0$ and $f(M) > 0$ with $f(m) \neq 0 \quad \forall \quad m \in (0, M)$

$f(0) > 0 \quad \Rightarrow \quad u(0,x) - p > \frac{w+\phi}{\pi(0,x)} \quad$ or $\quad u(0,x) - p > \underline{n}$

$f(M) > 0 \quad \Rightarrow \quad u(M,x) - p > \frac{w+\phi}{\pi(M,x)} \quad$ or $\quad u(M,x) - p > \overline{n}$

$f(m) \neq 0 \quad \Rightarrow \quad u(m,x) - p \neq \frac{w+\phi}{\pi(m,x)} \quad \forall \quad m \in (0, M)$

Shape of curve I and curve III will be as shown below in figure 4.

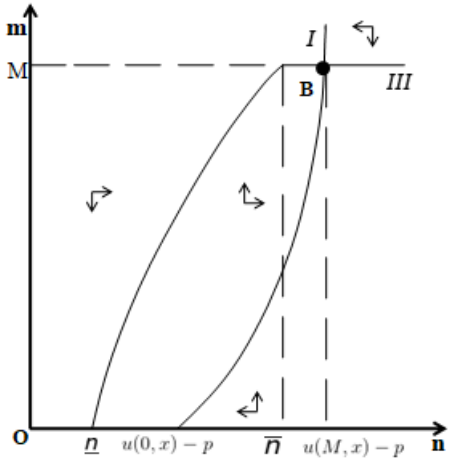


Figure 4: Only Two Stable (Zero and Full) Equilibrium Case

2b. Only Two Stable Equilibrium: Zero and Non-Full Participation

One stable equilibrium will be zero participation equilibrium. Another stable equilibrium with lower than full participation will mean that $f(m) = 0$ occur only once in $n \in (\underline{n}, \overline{n})$. This will happens only when $f(0) > 0$ with $f(M) < 0$ and $f'(m) < 0 \quad \forall m \in [0, M]$. If sufficiency condition holds, i.e. $\pi_{mm} \leq 0$, and $f'(0) < 0$ then $f'(m) < 0 \quad \forall m \in [0, M]$. Thus required conditions for one stable and one unstable equilibria will be as follows

$f(0) > 0 \Rightarrow u(0,x) - p > \frac{w+\phi}{\pi(0,x)} \quad$ or $\quad u(0,x) - p > \underline{n}$

$f(M) < 0 \Rightarrow u(M,x) - p < \frac{w+\phi}{\pi(M,x)} \quad$ or $\quad u(M,x) - p < \overline{n}$

$f'(0) < 0 \Rightarrow u_m(0,x) < \frac{-\pi_m(0,x)(u(0,x)-p)}{\pi(0,x)}$

$\pi_{mm}(m,x) \leq 0 \quad \forall \quad m \in [0, M]$

Above conditions will ensure that the graph of curves I and III should look as shown in figure 5

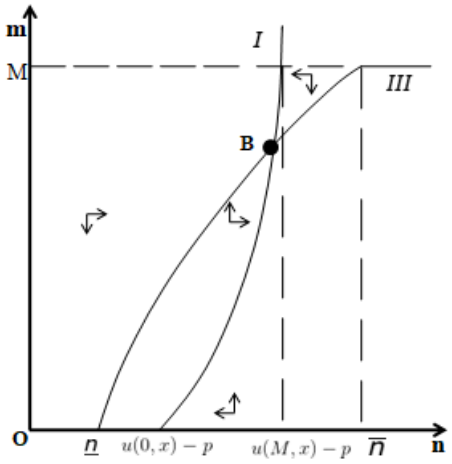


Figure 5: Two Stable (Zero and Non-Full) Equilibrium Case

3. Two Equilibrium: One Stable and One Unstable

Two equilibrium with one stable and another unstable will occur when curves I and III are tangent to each other at one point instead of crossing each other. But because of shape of curve I and III, it will never happen when $f(0) > 0$. This case only arise when $f(0) < 0$.

When $f(0) < 0$, we will get only one stable and unstable equilibrium if $f(M) < 0$ and $f'(m)$ will be zero at only one point where $f(m)$ is itself zero. That is, curve I and III are tangent to each other at one point for $n \in (\underline{n}, \overline{n})$. Let the number of sellers at the point of tangent (or at unstable equilibrium) is denoted by m^*. Now, the conditions under which one stable and one unstable equilibrium exist will be

$f(0) < 0 \quad \Rightarrow \quad u(0,x) - p < \frac{w+\phi}{\pi(0,x)} \qquad \text{or} \qquad u(0,x) - p < \underline{n}$

$f(M) < 0 \quad \Rightarrow \quad u(M,x) - p < \frac{w+\phi}{\pi(M,x)} \qquad \text{or} \qquad u(M,x) - p < \overline{n}$

$f'(m^*) = 0$ where m^* is given by $f(m^*) = 0$

Third condition implies that for a $m^* \in [0, M]$ such that it satisfy following two conditions -

$u(m^*,x) - p = \frac{w+\phi}{\pi(m^*,x)} \quad \& \qquad u_m(m^*,x) = \frac{-\pi_m(m^*,x)(u(m^*,x)-p)}{\pi(m^*,x)}$

If following condition holds then the graph of curve I and III will look like as shown in figure 6 -

We have discuss the conditions under which one and two equilibrium exist with seller competition on the platform. Three equilibrium (two stable and one unstable) will also exist under this scenario. In three equilibrium case, besides zero participation stable equilibrium, other stable equilibrium can serve full or less than full participation from seller side.

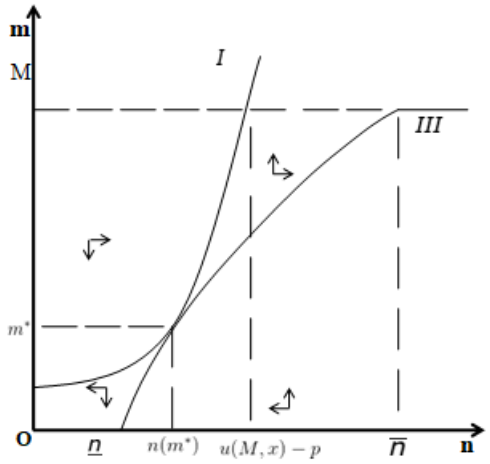


Figure 6: Unstable Equilibrium at Tangent Point

A2. Proof of Lemma

If assumption 2 holds, then the profit expression for unfavourable expectation is -

$$\Pi^{uf}(p,x) = p[u(M,x) - p] + M[\pi(0,x)\max(u(0,x) - p, 0) - \phi]$$

Now fix the x. If $p < u(0,x)$, seller subsidy strategy became feasible. Let us denoted price for seller subsidy case as p_1 and corresponding profit function as Π^1. If $p > u(0,x)$, buyer attraction strategy will be feasible. Let us denote price for this case as p_2 corresponding profit function as Π^2. From the first order condition of Π^{uf}, we will get prices for both the case for given x and is given by -

$$p_1 = \frac{u(M,x)}{2}; \qquad p_2 = \frac{u(M,x) - M\pi(0,x)}{2}$$

Now $p_1 < u(0,x) \Rightarrow \quad u(M,x) > 2u(0,x)$ and $p_2 > u(0,x) \Rightarrow \quad u(M,x) < 2u(0,x) + M\pi(0,x)$. Thus three possibilities arise-

- When $u(M,x) > 2u(0,x) + M\pi(0,x)$, only seller subsidy is feasible and optimal profit function is Π^1.
- When $u(M,x) < 2u(0,x)$, only buyer attraction strategy is feasible and optimal profit function is Π^2.
- When $2u(0,x) < u(M,x) < 2u(0,x) + M\pi(0,x)$, both seller subsidy and buyer attraction strategy is feasible and optimal profit function will be either Π^1 or Π^2 depending upon which strategy fetch higher profit.

The profit functions for seller subsidy and buyer attraction strategy are given by -

$$\Pi^1(x) = \frac{(u(M,x))^2}{4} - c(x) - M\phi; \qquad \Pi^2(x) = \frac{(u(M,x) - M\pi(0,x))^2}{4} + M\pi(0,x)u(0,x) - c(x) - M\phi$$

When $2u(0,x) < u(M,x) < 2u(0,x) + M\pi(0,x)$, platform will adopt seller attraction strategy for a given x if $\Pi^1 > \Pi^2$ and vice verse. Π^1 will be greater than Π^2 if $u(M,x) > 2u(0,x) + \frac{M\pi(0,x)}{2}$. Thus for unfavourable expectation platform, objective function can be written as -

$$\Pi^{uf}(x) = \begin{cases} \frac{(u(M,x))^2}{4} - c(x) - M\phi & \text{if } u(M,x) \geq 2u(0,x) + \dfrac{M\pi(0,x)}{2} \\ \frac{(u(M,x) - M\pi(0,x))^2}{4} + M\pi(0,x)u(0,x) - c(x) - M\phi & \text{if } u(M,x) \leq 2u(0,x) + \dfrac{M\pi(0,x)}{2} \end{cases}$$

□

A3. Proof of Proposition

Let us assume that x_f, x_1, and x_2 are uniquely defined. To get the relation between x_f and x_1, we will put x_1 in equation 1 which determine the level of x_f. That is, if the $\frac{\partial \Pi^f}{\partial x}$ at x_1 is positive, then it means $x_1 < x_f$ and vice verse. The similar analysis will give relation between x_f and x_2 when we put x_2 in place of x_f in equation 1.

$$\left.\frac{\partial \Pi^f}{\partial x}\right|_{x=x_1} = \frac{[u(m_f,x_1) + m_f\pi(m_f,x_1)][u_x(m_f,x_1) + m_f\pi_x(m_f,x_1)]}{2} - c'(x_1)$$

On putting the value of $c'(x_1)$, we will get -

$$\begin{aligned} &\frac{[u(m_f,x_1) + m_f\pi(m_f,x_1)][u_x(m_f,x_1) + m_f\pi_x(m_f,x_1)]}{2} - c'(x_1) = \\ &\frac{1}{2}\Big[[u(m_f,x_1) + m_f\pi(m_f,x_1)][u_x(m_f,x_1) + m_f\pi_x(m_f,x_1)] - u(M,x_1)u_x(M,x_1)\Big] \end{aligned} \tag{22}$$

Since $m_f < M$, exact sign of above expression is ambiguous even when x and m are complementary. We can also write the expression in square bracket as $G(m_f,x_1)G(m_f,x_1) - u(M,x_1)u_x(M,x_1)$. So, whether $x_1 < x_f$ or $x_1 > x_f$ will in the end depend upon $G(m_f,x_1)G(m_f,x_1) - u(M,x_1)u_x(M,x_1) > 0$ or $G(m_f,x_1)G(m_f,x_1) - u(M,x_1)u_x(M,x_1) < 0$ respectively. The relation between x and m still play an important role. When x and m are complementary, chances of $G(m_f,x_1)G(m_f,x_1) - u(M,x_1)u_x(M,x_1)$ being positive is higher as compare to when x and m are substitute. In that sense, sign of $\pi_x(m,x)$ is a crucial factor in determining the sign of above expression, but that is not all that determine the sign. Another factor which play an important role in seller subsidy case is that the total surplus generated on the buyer side is higher in seller subsidy strategy than the favourable platform. If this higher utility on buyer side and its response with respect to x is high

enough, then unfavourable platform with seller subsidy strategy could invest higher in first party content even when x and m are complementary.

Similarly, when we put x_2 in the first order condition for x_f we get

$$\left.\frac{\partial \Pi^f}{\partial x}\right|_{x=x_2} = \frac{[u(m_f,x_2)+m_f\pi(m_f,x_2)][u_x(m_f,x_2)+m_f\pi_x(m_f,x_2)]}{2} - c'(x_2)$$

$$= \frac{(u(m_f,x_2)+m_f\pi(m_f,x_2))(u_x(m_f,x_2)+m_f\pi_x(m_f,x_2)) - (u(M,x_2)-M\pi(0,x_2))(u_x(M,x_2)-M\pi_x(0,x_2))}{2}$$
$$- M[\pi_x(0,x_2)u(0,x_2)+\pi(0,x_2)u_x(0,x_2)]$$

$$= \frac{1}{2}\Big[[G(m_f,x_2)G_x(m_f,x_2)-u(M,x_2)u_x(M,x_2)]+M\pi_x(0,x_2)[u(M,x_2)-2u(0,x_2)-M\pi(0,x_2)]$$
$$+ M\pi(0,x_2)[u_x(M,x_2)-2u_x(0,x_2)]\Big]$$

Now, in buyer attraction strategy $G(m_f,x_2)G_x(m_f,x_2)-u(M,x_2)u(m,x_2)$ will not alone decide the relation between x_f and x_2. Since buyer attraction strategy is feasible only when $u(M,x) < 2u(0,x)+\frac{M\pi(0,x_2)}{2}$, so $u(M,x_2)-2u(0,x_2)-M\pi(0,x_2)<0$. When x and m are substitute then $u_x(M,x_2)-2u_x(0,x_2)<0$ but the sign will be ambiguous if x and m are complement. But if relation between x and m is not very strong for buyers, then we can say that $u_x(M,x_2)-2u_x(0,x_2)<0$ regardless of x and m are complement or substitute. And we know that $\pi_x(m,x)$ will be positive (negative) when x and m are complement (substitutes). Thus for a given sign of $G(m_f,x_2)G_x(m_f,x_2)-u(M,x_2)u(m,x_2)$, if relation between x and m are not very strong for buyers, then of being x and m complementary will leads to higher investment in first party content in comparison to x and m being substitute. This is in contrary to earlier established results.