Seller Competition and Platform Entry into Sellers' Product Space*

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Abstract

Digital marketplaces often sell their own products alongside those of third-party sellers. This paper studies how that dual role changes participation, access fees, and surplus when sellers compete on the platform. It compares a pure intermediary with a platform that enters as one additional seller under monopoly and under two forms of platform competition. The platform-owned seller receives no preferential treatment, so the analysis isolates the pricing and participation effects of entry. Under monopoly, entry always raises seller participation and, under an elasticity condition, also raises buyer participation. With an additional outside-option condition, buyers, sellers, and the platform all gain. Under symmetric single-homing platform competition, entry continues to lower seller fees and raise seller surplus, but its effects on buyers and platform profit depend on the competitive environment. In the baseline Hotelling framework, buyer fees rise and buyer surplus falls. When buyers instead value the platform-owned product as a higher-quality option, entry can lower buyer fees and raise platform profit over an intermediate range of parameters. Seller gains are therefore robust in the model, while buyer and platform gains depend on demand, quality, and market structure.

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1 Introduction

Many digital marketplaces play two roles at once. They connect buyers with independent sellers, but they also sell products of their own. Amazon combines a marketplace with first-party retail and private labels; app stores distribute third-party applications while offering platform-owned applications; and search or travel platforms may compete directly with the firms they list. This dual role can intensify product-market competition and make the platform more attractive to buyers. It can also change the fees faced by independent sellers and give the platform a reason to steer demand toward its own product. The welfare effect of platform entry is therefore not determined by entry alone. It depends on how entry changes participation, access prices, and competition on both sides of the market.

This paper studies one part of that problem: how a platform's entry into the sellers' product space changes equilibrium when independent sellers already compete with one another. Seller participation creates value for buyers through variety and availability, but it also divides demand among more firms and lowers each seller's profit per buyer. The platform must balance these forces when it chooses the membership fees charged to buyers and sellers. Entry adds a further source of revenue—the operating profit of the platform-owned seller—and changes that balance.

The analysis compares a pure platform with a hybrid platform that owns one seller on its marketplace. The platform-owned seller competes on the same terms as every other seller. In particular, the model rules out preferential ranking, discriminatory access, and the use of privileged data. This assumption is deliberate. It provides a benchmark in which any change in participation or surplus comes from the platform's pricing incentives and its direct stake in product-market profit, rather than from self-preferencing.

The monopoly results show how the two-sided feedback loop can make entry mutually beneficial. Because the platform-owned seller does not pay a seller membership fee, entry increases the marginal return to seller participation. Seller participation consequently rises. Buyer participation also rises when the gain that additional sellers create for buyers is large enough relative to the reduction in each seller's profit. Higher buyer participation then supports seller revenue despite more intense competition. Under this elasticity condition, both buyer and seller surplus increase. If the platform-owned seller's outside option is at least as large as the intrinsic value that an independent seller obtains from joining, the platform's profit rises as well. Platform entry then produces a win-win-win outcome.

Competition between platforms changes the mechanism. With single-homing and full market

coverage, aggregate buyer and seller participation is fixed. A lower fee cannot expand the overall market; it can only affect how surplus is divided. In the baseline Hotelling framework, entry by both platforms lowers seller fees and raises seller surplus, but it raises buyer fees and reduces buyer surplus. Each platform benefits only when the additional product-market return is large enough to offset the change in membership-fee revenue.

The paper also considers an alternative competitive framework in which buyers differ both in their platform location and in their taste for quality. Buyers perceive the platform-owned product as a higher-quality option. Entry again lowers the fee paid by independent sellers. Its effect on buyers is now conditional: buyer fees fall when the perceived quality advantage is sufficiently large relative to the strength of the seller-side network effect. Platform profit rises when that network effect is strong enough. An intermediate parameter range therefore restores the win-win-win result under platform competition.

These findings separate a robust result from two conditional ones. Independent sellers benefit from platform entry throughout the competitive environments studied here because their membership fee falls. Buyer welfare and platform profitability are more sensitive to demand and market structure. This distinction helps explain why theoretical work on hybrid marketplaces reaches different welfare conclusions: the answer depends on whether own-product entry expands participation, changes variety, alters seller fees, or steers demand.

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 develops the monopoly model and compares participation, fees, profit, and surplus under pure and hybrid operation. Section 4 studies competition between symmetric single-homing platforms. Section 5 introduces horizontal and vertical differentiation on the buyer side. Section 6 concludes and discusses policy implications. The appendix contains the proofs.

2 Literature Review

The starting point is the literature on pricing in two-sided markets. Rochet and Tirole [2006] and Armstrong [2006] show that the relevant object is the structure of prices across the two sides, because participation by one group changes the value offered to the other. A platform may therefore subsidize the side that is more important for attracting the other group. This paper uses that logic but adds a negative within-group effect among sellers: an additional seller attracts buyers while reducing the operating profit earned by each incumbent seller.

Several papers examine that tradeoff directly. Hagiu [2009] studies product variety and platform pricing when producers compete, while Belleflamme and Toulemonde [2009] shows how positive cross-group effects can coexist with negative effects within a group. Galeotti and Moraga-González [2009] analyzes intermediation with differentiated sellers, and Belleflamme and Peitz [2019] shows that a platform may restrict seller participation to protect seller margins. The pure-platform benchmark in this paper is closest to Belleflamme and Peitz [2019]; the additional question is how the platform changes its access prices once it becomes one of the competing sellers.

Platform ownership of content or products has also been studied as a solution to coordination and organizational problems. Hagiu and Spulber [2013] show that first-party content can overcome unfavorable expectations and help a platform bring both sides on board. Hagiu and Wright [2015] compare marketplace and reseller modes and relate the choice to information, control, and incentive tradeoffs. Jiang et al. [2011] studies a marketplace's incentive to use information from third-party sales when deciding whether to introduce its own product. These papers establish that operating on the seller side can do more than generate direct sales: it can alter participation, control, and the terms offered to independent firms.

The empirical evidence points to both selection and competitive effects. Zhu and Liu [2018] find that Amazon is more likely to enter successful product spaces and that affected third-party sellers subsequently reduce their activity on the platform. Zhu [2019] emphasizes that entry can create value for an ecosystem while also threatening complementors. This evidence motivates the policy concern, but it does not by itself identify whether the harm comes from direct competition, the use of seller information, or preferential treatment. The benchmark here removes the latter two channels and isolates the fee and participation effects of direct entry.

Recent theory treats the platform's marketplace-plus-retail structure as a hybrid or dual mode. Etro [2021] studies product selection by a marketplace that can earn commissions from third-party sales or retail products itself. Anderson and Bedre-Defolie [2022] shows that hybrid operation can help consumers when it lowers prices, but can hurt them when heterogeneous tastes and seller fees reduce variety. Hagiu et al. [2022] distinguishes the welfare effects of platform selling from those of product imitation and self-preferencing, and shows why restrictions on particular conduct can dominate a blanket ban on dual mode. These contributions make clear that platform ownership and discriminatory treatment are separate economic choices.

Seller entry and the platform's fee response are central in the newer hybrid-platform literature. Anderson and Bedre-Defolie [2024] models a dominant platform product alongside a monopolistically

competitive fringe and finds that hybrid operation can raise third-party fees, reduce seller variety, and harm consumers. By contrast, Dendorfer [2024] finds that first-party selling can reduce double marginalization and raise welfare even when self-preferencing is possible. The survey by Kittaka et al. [2023] likewise concludes that the welfare effects of first-party selling and self-preferencing vary with the conduct and market environment being studied.

This paper differs from that literature in three ways. First, it models access through membership fees on both sides rather than only a commission on transactions. Second, it compares monopoly with two forms of competition between single-homing platforms. Third, it assumes equal treatment of the platform-owned and independent sellers. The contribution is therefore a clean participation benchmark: own-product entry always improves the position of sellers in the environments studied, but its effects on buyers and platform profit depend on whether entry expands the market, changes access fees in a covered market, or adds a sufficiently valued quality option.

3 Monopoly Platform: The Setup

In this section, we compare and contrast buyers' and sellers' participation on a monopoly platform under two alternative scenarios. In the first scenario, the monopoly platform is a pure platform i.e. the platform is only engaged in the process of connecting buyers and sellers. In this case, the platform earns its profit from the membership fee it charges buyers and sellers only. In the second scenaio, the platform enters into the sellers' product space i.e. apart from connecting buyers and sellers, the platform competes with the sellers and acts as one of the sellers on the platform. Here, the platform has two sources of income the membership fee it charges buyers and sellers and its own sales revenue.

Now, let us denote the number of buyers and sellers on the platform by n_b and n_s respectively. Also, the membership fees charged to each buyer and seller are given by m_b and m_s respectively. We have assumed that there exists indirect externalities from buyers to sellers and from sellers to buyers. Also, we have focused on the situation where the within group effect is absent for the buyers, but there is negative within group externality effect for the sellers. This is a common feature of platforms like Amazon, Flipkart, and shopping malls where buyers do not care about other buyers but sellers compete among themselves. Let us define the net surplus of each buyer and seller as

$$v_b = r_b + n_s u\left(n_s\right) - m_b$$

$$v_s = r_s + n_b \pi\left(n_s\right) - m_s$$

where r_b and r_s are the intrinsic utilities derived by buyers and sellers from being on the platform. $u(n_s)$ is the per seller utility derived by each buyer. $\pi(n_s)$ is the per buyer profit earned by each seller. Thus $n_b\pi(n_s)$ is the total gain from trade for each seller. The stages of the game are as follows

Stage 1: The monopoly platform sets membership fee for both the groups i.e. m_b and m_s; which are observable and verifiable by all.

Stage 2: After observing m_b and m_s, the buyers and sellers simultaneously decide to join or not join the platform.

Stage 3: Sellers on the platform engage in oligopoly competition.

We solve the game by backward induction. As in Belleflamme & Peitz (2019), we assume that the buyers and sellers of mass Z have an outside option (or reservation utility) which is uniformly distributed on $[0, Z]$, where Z is very large. Therefore, only those buyers and sellers who fetch a higher utility by joining the platform in comparison to the outside option will join the platform. Thus, the equilibrium numbers of buyers and sellers on the platform are given by

$$n_b = r_b + n_s u(n_s) - m_b; \quad n_s = r_s + n_b\pi(n_s) - m_s$$

From the above equations, the membership fee for each buyer and seller can be written in terms of buyer and seller participation, given by

$$m_b = r_b + n_s u(n_s) - n_b; \quad m_s = r_s + n_b\pi(n_s) - n_s \tag{1}$$

3.1 Pure Monopoly Platform

The pure platform's only source of revenue is from the membership fee it charges to each buyer and seller. Therefore, the profit expression for the pure monopoly platform is given by

$$\Pi = n_b(m_b - f_b) + n_s(m_s - f_s)$$

, where f_b and f_s are the costs of serving each buyer and seller respectively. Now, by using equation 2.1 , we can express the profit function of the platform solely in terms of n_b and n_s. Thus, the problem of the platform in stage 1 is to choose buyers' and sellers' participation (n_b and n_s) such that it maximizes its profit. This buyers' and sellers' participation in turn will decide the membership fee charged by the platform to each buyer and seller. Therefore, the problem of the

platform in stage 1 can be written as follows

$$\max_{n_b,n_s} \Pi = n_b \left[r_b + n_s u\left(n_s\right) - n_b - f_b\right] + n_s \left[r_s + n_b \pi\left(n_s\right) - n_s - f_s\right]$$

The first order conditions for the number of buyers and sellers are

$$\begin{aligned} \frac{\partial \Pi}{\partial n_b} &= r_b - 2n_b - f_b + \left[u\left(n_s\right) + \pi\left(n_s\right)\right] n_s = 0 \\ \frac{\partial \Pi}{\partial n_s} &= r_s - 2n_s - f_s + \left[u\left(n_s\right) + \pi\left(n_s\right)\right] n_b + \left[u'\left(n_s\right) + \pi'\left(n_s\right)\right] n_b n_s = 0 \end{aligned} \tag{2}$$

Let the equilibrium number of buyers and sellers be denoted by n_b^* and n_s^* respectively. With n_b^* and n_s^* as the equilibrium number of buyers and sellers on the pure monopoly platform, the membership fee for each buyer and seller will be given by

$$\begin{aligned} m_b^* &= r_b + n_s^* u\left(n_s^*\right) - n_b^* = f_b + n_b^* - n_s^* \pi\left(n_s^*\right) \\ m_s^* &= r_s + n_b^* \pi\left(n_s^*\right) - n_s^* = f_s + n_s^* - n_b^* u\left(n_s^*\right) - n_b^* n_s^* \left[u'\left(n_s^*\right) + \pi'\left(n_s^*\right)\right] \end{aligned} \tag{3}$$

where m_b^* and m_s^* are the equilibrium membership fee charged from each buyer and seller respectively by the pure platform. Equations 2.2 and 2.3 will characterize the equilibrium buyers' and sellers' participation and the membership fee charged by the pure monopoly platform to each buyer and seller.

2.2.2 Monopoly Platform Enters into sellers' Product Space

When the platform enters into the sellers' product space, the platform itself acts as one of the sellers on the platform and competes with other sellers. The profit of the seller owned by the platform will be an another source of profit for the platform. Here, we have assumed that the seller owned by the platform will also have an outside option and will operate on the platform only if the utility earned by joining the platform is higher than the outside option. Thus, the seller owned by the platform is the same as any other seller on the platform except for two distinctions. First, the profit of this seller will be a part of the overall profit of the platform and second, this seller does not have to pay a membership fee. Now, the profit expression of the platform when it enters into the sellers' product space can be written as

$$\Pi^o = n_b \left(m_b - f_b\right) + n_s \left(m_s - f_s\right) + \left(n_b \pi\left(n_s\right) - m_s\right)$$

From the equation 2.1, we can express m_b and m_s in terms of buyers' and sellers' participation and write the platform's problem in stage 1 solely in terms of n_b and n_s. Thus, the problem of the

platform in stage 1 can be written as follows

$$\max_{n_b,n_s} \Pi^o = n_b \left[r_b + n_s u\left(n_s\right) - n_b - f_b\right] + n_s \left[r_s + n_b \pi\left(n_s\right) - n_s - f_s\right] + \left(n_s - r_s\right)$$

Now, the first order condition will be given by

$$\begin{aligned} \frac{\partial \Pi^o}{\partial n_b} &= r_b - 2n_b - f_b + \left[u\left(n_s\right) + \pi\left(n_s\right)\right] n_s = 0 \\ \frac{\partial \Pi^o}{\partial n_s} &= r_s - 2n_s - f_s + \left[u\left(n_s\right) + \pi\left(n_s\right)\right] n_b + \left[u'\left(n_s\right) + \pi'\left(n_s\right)\right] n_b n_s + 1 = 0 \end{aligned} \tag{4}$$

Let us assume that (n_b^o, n_s^o) is the unique solution of the system of equations given in 2.4 where n_b^o and n_s^o represent equilibrium participation of buyers and sellers respectively when platform enter into the sellers' product space. Now using equation 2.1, the equilibrium membership fees for each buyer and seller are given by

$$\begin{aligned} m_b^o &= r_b + n_s^o u\left(n_s^o\right) - n_b^o = f_b + n_b^o - n_s^o \pi\left(n_s^o\right) \\ m_s^o &= r_s + n_b^o \pi\left(n_s^o\right) - n_s^o = f_s + n_s^o - n_b^o u\left(n_s^o\right) - n_b^o n_s^o \left[u'\left(n_s^o\right) + \pi\left(n_s^o\right)\right] - 1 \end{aligned} \tag{5}$$

where m_b^o and m_s^o are the equilibrium membership fees for each buyer and seller respectively when the platform enters into the sellers' product space. Equations 2.4 and 2.5 will characterize the equilibrium buyers' and sellers' participation and the membership fee charged by the platform from each buyer and seller in the case in which the monopoly platform enters into sellers' product space.

2.2.3 Comparison of Buyers and Sellers Participation in Both Scenarios

The set of first order conditions given by equation 2.2 and 2.4 will provide us buyers' and sellers' participation in each of the cases. From equations 2.2 and 2.4, it is straightforward to observe the following:

$$\left.\frac{\partial \Pi^o}{\partial n_b}\right|_{\left(n_b^*, n_s^*\right)} = 0 \quad \text{and} \quad \left.\frac{\partial \Pi^o}{\partial n_s}\right|_{\left(n_b^*, n_s^*\right)} = 1$$

Now,

$$\left.\frac{\partial \Pi^o}{\partial n_s}\right|_{\left(n_b^*, n_s^*\right)} > 0 \Rightarrow n_s^o > n_s^* \tag{6}$$

Thus, in the second scenario, the platform will a larger number of sellers in the equilibrium whereas the number of buyers in the second equilibrium can either be less or more than, but not

equal to, n_b^*. This can easily be shown in the following manner. We know that $\frac{\partial \Pi^o}{\partial n_b}\Big|_{(n_b^*, n_s^*)} = 0$ and if $n_b^o = n_b^*$, then the number of sellers in the second scenario must be equal to the sellers' participation in the pure monopoly case, i.e. $n_s^o = n_s^*$, which is in contradiction to our finding that $n_s^o > n_s^*$. Since we know that the sellers' participation will be higher in the second scenario, the buyers' participation in the second case will be higher or lower depending on whether $\frac{\partial \Pi^\circ}{\partial n_b}$ is increasing or decreasing as the number of sellers on the platform increases.

$$\frac{\partial^2 \Pi^o}{\partial n_s \partial n_b}\Big|_{\left(\frac{\partial \Pi^o}{\partial n_b}=0\right)} = u(n_s) + \pi(n_s) + n_s\left[u'(n_s) + \pi'(n_s)\right]$$

If the above expression is positive, then the equilibrium buyers' participation in the second scenario will be higher or vice versa. The expression in the right hand side of the above equation can also be written as

$$u(n_s) + \pi(n_s) + n_s\left[u'(n_s) + \pi'(n_s)\right] = u(n_s)\left[1 + \frac{n_s}{u(n_s)}\frac{\partial u(n_s)}{\partial n_s}\right] + \pi(n_s)\left[1 + \frac{n_s}{\pi(n_s)}\frac{\partial \pi(n_s)}{\partial n_s}\right]$$

Now, let us define $\frac{n_s}{u(n_s)}\frac{\partial u(n_s)}{\partial n_s} = \eta_u$; and $\frac{n_s}{\pi(n_s)}\frac{\partial \pi(n_s)}{\partial n_s} = \eta_\pi$ where η_u can be interpreted as the elasticity of per seller utility of each buyer with respect to sellers' participation. Similarly, η_π can be interpreted as elasticity of per buyer sellers' profit with respect to sellers' participation. Thus when $u(n_s) + \pi(n_s) + n_s\left[u'(n_s) + \pi'(n_s)\right] = u(n_s)\left[1 + \eta_u\right] + \pi(n_s)\left[1 + \eta_\pi\right] > 0$, then buyers' participation on the platform will be higher in second scenario. This condition can also be written as

$$\frac{u(n_s)}{\pi(n_s)} > \frac{-(1+\eta_u)}{(1+\eta_\pi)} \tag{7}$$

Since, for buyers, the gain from trade will increase with increase in sellers participation,

$$\frac{\partial n_s u(n_s)}{\partial n_s} > 0$$

$$\Rightarrow \quad \eta_u > -1 \quad \text{or} \quad 1 + \eta_u > 0$$

Also, $u(n_s)$ and $\pi(n_s)$ are both strictly positive quantities. Therefore, if $|\eta_\pi| < 1$ holds, the condition 2.7 is always going to be satisfied. Thus, the sufficiency condition for higher buyers' participation in the second scenario is $|\eta_\pi| < 1$.

The number of buyers on the platform will depend on sellers' participation and membership fee charged to the buyers. If membership fee for buyers will decrease with sellers' participation

then buyers participation will definitely increase. This is the sufficiency condition for higher buyers' participation ($|\eta_\pi| < 1$).

An Example

Let us take an example where $u(n_s) = n_s$ and $\pi(n_s) = \frac{1}{\sqrt{n_s}}$.

$$\eta_u = 1 \quad \text{and} \quad \eta_\pi = -\frac{1}{2}$$

Condition 2.7 implies

$$n_s > (4)^{\frac{2}{3}}$$

That is, whenever the sellers' participation is more than or equal to 3 ($n_s \geq 3$) on the platform, then the condition 2.7 will be satisfied and buyers participation will increase with the entry of platform into the product space.

Lemma 2.1: Suppose condition 2.7 holds, then in equilibrium, the monopoly platform serves higher number of buyers and sellers when it enters into the sellers' product space in comparison to when it remains a pure monopoly platform i.e. $n_b^o > n_b^*$ and $n_s^o > n_s^*$.

3.2 Platform's Profit and Surplus of Buyers and Sellers

Now, we compare the profits generated by the monopoly platform and the surplus earned by buyers and sellers under the two alternative scenarios - pure monopoly platform and when the monopoly platform enters into sellers' product space.

Platform's Profit

The profit expressions of the monopoly platform for any given n_b and n_s when the platform is a pure platform and when the platform enters into sellers' product space are, respectively

$$\Pi = n_b(m_b - f_b) + n_s(m_s - f_s) \text{ and}$$
$$\Pi^o = n_b(m_b - f_b) + n_s(m_s - f_s) + (n_b\pi(n_s) - m_s)$$

where Π and Π° are the profit earned by the pure monopoly platform and when monopoly platform itself act as one of the seller respectively. On comparing the above two equations, we can say that for any given $(n_b, n_s), \Pi^o > \Pi$, if $(n_b\pi(n_s) - m_s) > 0$.

All the sellers who joined the platform have an outside option (or reservation utility). Let us denote the outside option of any given seller by z_s. Any seller will join the platform only if the utility earned by joining the platform is greater than or equal to the outside option, i.e. $v_s \geq z_s$, where equalitys hold only for the marginal seller (the last seller who is indifferent between joining or not joining the platform). $v_s \geq z_s$ implies that $r_s + n_b\pi(n_s) - m_s \geq z_s$ for all the sellers on the platform. Thus, for all the sellers who join the platform

$$n_b\pi(n_s) - m_s \geq z_s - r_s \quad \text{where} \quad z_s \in [0, Z]$$

It follows that if the seller owned by the platform is such that $z_s - r_s \geq 0$, i.e. the outside option of the seller owned by the platform is higher than the intrinsic utility it gets from joining the platform, then it follows that $n_b\pi(n_s) - m_s \geq 0$.[1] Thus, whenever the platform owns a seller whose outside option is greater than the intrinsic utility, then $\Pi^o \geq \Pi$ for any given pair of (n_b, n_s). Now, since buyers' and sellers' participation is different in both the scenarios, we have to prove that Π^o at (n_b^o, n_s^o) is higher than Π at (n_b^*, n_s^*) if $z_s \geq r_s$ for the platform-owned seller.

Lemma 2.2: If the seller owned by the platform has an outside option which is higher than the intrinsic utility of joining the platform, then in the equilibrium, the monopoly platform which enters into the sellers' product space will earn a higher profit than the pure monopoly platform.

The proof of lemma 2.2 is given in the appendix B.1. The necessary condition for $\Pi^o(n_b^o, n_s^o) > \Pi(n_b^*, n_s^*)$ is that the seller owned by the platform must have an outside option which is higher than the intrinsic utility obtained from joining the platform. This condition means that the seller which is owned by the platform has a very good outside option and if the platform does not own this seller, it will not be very easy for the platform to attract this seller on the platform.

Surplus of Buyers and Sellers

Since any buyer or seller will join the platform if and only if the utility obtained by joining the platform is higher than the outside option available to buyers and sellers[2]. Therefore, if more number of buyers and sellers join the platform, it means that higher surplus is generated by all the buyers and sellers on the platform. We know that when the monopoly platform enters into the

[1] Since the store which is owned by the platform does not have to pay the membership fee, the participation condition for that seller is $n_b\pi(n_s) \geq z_s - r_s$. But we impose a more stringent condition as for any given z_s, if $n_b\pi(n_s) - m_s \geq z_s - r_s$ then $n_b\pi(n_s) \geq z_s - r_s$ will hold for sure.

[2] In this model, we have assumed that the outside option available to buyers and sellers is not affected by the activity of the platform.

sellers' product space, it servea a higher number of sellers on the platform. Also, if condition 2.7 holds, then $n_b^o > n_b^*$. This implies that, if condition 2.7 holds, then the surplus generated by the buyers and sellers is higher when a monopoly platform enters into sellers' product space.

Lemma 2.3: Suppose that the condition 2.7 holds. Then, the entry of a monopoly platform into the sellers' product space will lead to a higher surplus for both buyers and sellers on the platform compared to that of a pure monopoly platform.

From lemma 2.2 and lemma 2.3, the following proposition follows

Proposition 2.1: If the outside option of the seller owned by the platform is higher than the intrinsic utility it obtains from joining the platform and condition 2.7 holds, then the entry of the monopoly platform into the sellers' product space is beneficial for all - buyers, sellers and the platform itself, in comparison to the case of the pure monopoly.

3.2.1 Discussion on membership fees charged from buyers and sellers

Membership fees charged from each buyer and each seller, when monopoly platform is a pure platform are given by equation 2.3

$$m_b = r_b + n_s^* u\left(n_s^*\right) - n_b^* = f_b + n_b^* - n_s^* \pi\left(n_s^*\right)$$
$$m_s = r_s + n_b^* \pi\left(n_s^*\right) - n_s^* = f_s + n_s^* - n_b^* u\left(n_s^*\right) - n_b^* n_s^* \left[u'\left(n_s^*\right) + \pi\left(n_s^*\right)\right]$$

And, when the platform enters into the sellers' product space, the membership fees it charges each buyer and each seller are given by equation 2.5

$$m_b^o = r_b + n_s^o u\left(n_s^o\right) - n_b^o = f_b + n_b^o - n_s^o \pi\left(n_s^o\right)$$
$$m_s^o = r_s + n_b^o \pi\left(n_s^o\right) - n_s^o = f_s + n_s^o - n_b^o u\left(n_s^o\right) - n_b^o n_s^o \left[u'\left(n_s^o\right) + \pi\left(n_s^o\right)\right] - 1$$

Membership Fee Charged from the Buyer

From equation 2.3 and 2.5 , we know that the value of m_b and m_b^o will be the same for any given pair of (n_b, n_s). But we know that in equilibrium, buyers' and sellers' participation are different in the case of pure monopoly platform and in the case in which the monopoly platform act as one of the seller. That is, n_b^* is different from n_b^o and n_s^* is different from n_s^o. If condition 2.7 holds, then both buyers' and sellers' participation are higher when the platform enters into the sellers' product space; i.e $n_b^o > n_b^*$ and $n_s^o > n_s^*$. But with an increase in n_b, the membership fees for the buyer will increase whereas with an increase in $n_s, \pi\left(n_s\right)$ will decrease. Hence, the overall impact of an

increase in (n_b, n_s) is ambiguous. Now, differentiating $m_b = f_b + n_b - n_s\pi(n_s)$, we get

$$d(m_b) = d(n_b) - n_s \frac{\partial \pi(n_s)}{\partial n_s} \cdot d(n_s) - \pi(n_s) \cdot d(n_s)$$

$$\text{Thus,} \quad d(m_b) < 0 \Rightarrow \frac{dn_b}{dn_s} < \pi(n_s)[1 + \eta_\pi] \tag{8}$$

Thus, if the inequality 2.8 holds, then $m_b^o(n_b^o, n_s^o) < m_b(n_b^*, n_s^*)$, and vice versa.

Membership Fee Charged from the Seller

From equation 2.3 and 2.5 , for any given pair of (n_b, n_s), the membership fee charged from the seller by a monopoly platform which owns a seller on its platform is lower i.e. $m_s^o(n_b, n_s) < m_s(n_b, n_s)$ than the fees charged by a pure monopoly platform. This is because, for any given pair of (n_b, n_s)

$$m_s^o(n_b, n_s) = m_s(n_b, n_s) - 1 \quad \Rightarrow \quad m_s^o(n_b^*, n_s^*) < m_s(n_b^*, n_s^*)$$

However, when equation 2.7 holds then $n_b^o > n_b^*$ and $n_s^o > n_s^*$, so we cannot definitively say whether $m_s^o(n_b^o, n_s^o)$ is higher or lower than $m_s(n_b^*, n_s^*)$. If $m_s(n_b, n_s)$ is decreasing in (n_b^*, n_s^*), then we can definitely say that $m_s^o(n_b^o, n_s^o)$ will be lower than $m_s(n_b^*, n_s^*)$. This is a very stringent condition in the sense that even if this condition does not hold, $m_s^o(n_b^o, n_s^o)$ can be lower than $m_s(n_b^*, n_s^*)$.

$$d(m_s) = \pi(n_s)\, d(n_b) + (n_b\pi'(n_s) - 1)\, d(n_s)$$

Thus $d(m_s) < 0$ will imply that

$$\frac{dn_b}{dn_s} < \frac{1 - n_b\pi'(n_s)}{\pi(n_s)} \tag{9}$$

If equation 2.9 holds, then we can definitely say that $m_s^o(n_b^o, n_s^o) < m_s(n_b^*, n_s^*)$. But when equation 2.9 does not hold, we cannot conclusively say whether $m_s^o(n_b^o, n_s^o)$ is higher or lower than $m_s(n_b^*, n_s^*)$.

Lemma 2.4: Suppose conditions 2.8 and 2.9 hold, then the membership fee charged by the monopoly platform which acts as a seller in its own platform will be lower for buyers and sellers both in comparison to that charged by a pure monopoly platform.

4 Competition Among Platforms with Single Homing: The Setup

In this section, we consider the situation in which two platforms compete with each other to attract buyers and sellers on the platform. We assume that for some exogenous reason, each agent chooses to join only a single platform i.e. each agent will do single homing. As in the monopoly case, in this section, we compare and contrast the membership fees for buyers and sellers and the profit of each platform under the following two scenarios. In the first scenario, both the platforms are pure platforms. In the second scenario, both the platforms enter into the sellers' product space and compete with the sellers who decide to join their platform.

Two platforms, denoted by A and B , will facilitate the interaction between buyers and sellers. We assume that they are located at the extreme points of a unit interval. As in Belleflamme & Peitz (2010), we suppose that there are N_b and N_s number of buyers and sellers, respectively, which are located uniformly on the unit interval. The location of buyers and sellers determines their opportunity cost of visiting either platform. The opportunity cost increases linearly with the distance between platform and agent at the rate t_b and t_s for buyers and sellers respectively. Let $\left(n_b^i, n_s^i\right)$ denote the number of buyers and sellers who join the platform i, where $i \in \{A, B\}$. The utility of each buyer and each seller who join the platform i, denoted by v_b^i and v_s^i respectively, is determined the same way as in the case of the monopoly and is given by

$$\begin{aligned} v_b^i &= r_b + n_s^i u\left(n_s^i\right) - m_b^i \\ v_s^i &= r_s + n_b^i \pi\left(n_s^i\right) - m_s^i \end{aligned} \tag{10}$$

where m_b^i and m_s^i are the membership fees for the buyer and the seller respectively on the platform i. Now, suppose that the buyers' and sellers' participation on the platform are given by the Hotelling formulation

$$\frac{n_b^i}{N_b} = \frac{1}{2} + \frac{v_b^i - v_b^j}{2t_b}; \quad \frac{n_s^i}{N_s} = \frac{1}{2} + \frac{v_s^i - v_s^j}{2t_s} \quad \text{where} \quad i, j \in \{A, B\} \quad \text{and} \quad i \neq j$$

Using equation 10 in the above equation, buyers' and sellers' participation on the platform i can be expressed as

$$\begin{aligned} \frac{n_b^i}{N_b} &= \frac{1}{2} + \frac{n_s^i u\left(n_s^i\right) - \left(N_s - n_s^i\right) u\left(N_s - n_s^i\right)}{2t_b} - \frac{m_b^i - m_b^j}{2t_b} \\ \frac{n_s^i}{N_s} &= \frac{1}{2} + \frac{n_b^i \pi\left(n_s^i\right) - \left(N_b - n_b^i\right) \pi\left(N_s - n_s^i\right)}{2t_s} - \frac{m_s^i - m_s^j}{2t_s} \end{aligned}$$

given $n_b^i + n_b^j = N_b$ and $n_s^i + n_s^j = N_s$. This is because each buyer will join only a single platform. Now, let us define

$$\Delta u\left(n_s^i\right) = n_s^i u\left(n_s^i\right) - \left(N_s - n_s^i\right) u\left(N_s - n_s^i\right)$$
$$\Delta\pi\left(n_b^i, n_s^i\right) = n_b^i \pi\left(n_s^i\right) - \left(N_b - n_b^i\right) \pi\left(N_s - n_s^i\right)$$

Interpretation of $\Delta u\left(n_s^i\right)$ and $\Delta\pi\left(n_b^i, n_s^i\right)$ are as follows: $\Delta u\left(n_s^i\right)$ quantifies the difference between the buyer's net gain from trade on platform i vis-a-vis that on platform j when n_s^i is the sellers' participation on the platform i. Similarly, $\Delta\pi\left(n_b^i, n_s^i\right)$ measures the difference between the seller's net gain from trade on platform i and that on platform j when n_b^i and n_s^i are the buyers' and sellers' participation on platform i. So, the buyers' and sellers' participation on platform i can also be expressed as

$$\begin{aligned} \frac{n_b^i}{N_b} &= \frac{1}{2} + \frac{\Delta u\left(n_s^i\right)}{2t_b} - \frac{m_b^i - m_b^j}{2t_b} \\ \frac{n_s^i}{N_s} &= \frac{1}{2} + \frac{\Delta\pi\left(n_b^i, n_s^i\right)}{2t_s} - \frac{m_s^i - m_s^j}{2t_s} \end{aligned} \tag{11}$$

Now, let us define a few more things which will help us in deriving the membership fees for buyers and sellers

$$\Delta_s u\left(n_s^i\right) = \frac{\partial \Delta u\left(n_s^i\right)}{\partial n_s^i}; \quad \Delta_b \pi\left(n_b^i, n_s^i\right) = \frac{\partial \Delta\pi\left(n_b^i, n_s^i\right)}{\partial n_b^i}; \quad \Delta_s \pi\left(n_b^i, n_s^i\right) = \frac{\partial \Delta\pi\left(n_b^i, n_s^i\right)}{\partial n_s^i}$$
$$\phi = \frac{-2t_b}{N_b}\left(\Delta_s \pi\left(n_b^i, n_s^i\right) - \frac{2t_s}{N_s}\right) - \Delta_s u\left(n_s^i\right) \Delta_b \pi\left(n_b^i, n_s^i\right)$$

$\Delta_s u\left(n_s^i\right)$ measures the sensitivity of the differential in buyer's net gain from trade between platforms i and j with an increase in sellers' participation on platform i. Similarly, we can interpret $\Delta_b \pi\left(n_b^i, n_s^i\right)$ and $\Delta_s \pi\left(n_b^i, n_s^i\right)$ as a measure of sensitivity of sellers' net gain from trade between platform i and j with an increase in buyers' and sellers' participation, respectively. We will denote $\Delta_s u\left(n_s^i\right), \Delta_b \pi\left(n_b^i, n_s^i\right)$, and $\Delta_s \pi\left(n_b^i, n_s^i\right)$ by $\Delta_s u, \Delta_b \pi$, and $\Delta_s \pi$ respectively for simplicity. ϕ will be used in deriving the membership fees for buyers and sellers.

The stages of the game will be the same as it was in the monopoly case except that the buyers' and sellers' decision of whether to join or not in monopoly case will change to whether to join platform A or B. The stages of the game are as follows

Stage 1: Both the platforms will set membership fees for both the groups i.e. $\left(m_b^i, m_s^i\right)$ and $\left(m_b^j, m_s^j\right)$; which are observable and verifiable by all.

Stage 2: After observing the membership fees of both the platforms, the buyers and sellers simultaneously decide which platform to join.

Stage 3: Sellers on the platform engage in oligopoly competition.

We solve the game by backward induction. From equation 2.11 , we can easily verify that the explicit solution for buyers' and sellers' participation on platform i will also be a function of the membership fees of platform j i.e. $n_b^i\left(m_b^i, m_s^i, m_b^j, m_s^j\right)$ and $n_s^i\left(m_b^i, m_s^i, m_b^j, m_s^j\right)$.

4.1 Competition Among Pure Platforms

When both the platforms are pure platforms, the only source of revenue for the platforms are the membership fees they charge buyers and sellers. If we assume that the cost of serving each buyer and each seller is f_b and f_s respectively, then the profit expression for the pure platform i is given by

$$\Pi^i = \left(m_b^i - f_b\right) n_b^i + \left(m_s^i - f_s\right) n_s^i$$

Now, by using equation 2.11, we can express the profit function of platform i solely in terms of membership fees set by both the platforms, i.e. $\Pi^i\left(m_b^i, m_s^i, m_b^j, m_s^j\right)$. Thus, the problem of platform i in stage 1 can be expressed as follows

$$\max_{m_b^i, m_s^i} \Pi^i\left(m_b^i, m_s^i, m_b^j, m_s^j\right) = \left(m_b^i - f_b\right) n_b^i\left(m_b^i, m_s^i, m_b^j, m_s^j\right) + \left(m_s^i - f_s\right) n_s^i\left(m_b^i, m_s^i, m_b^j, m_s^j\right)$$

First order conditions for membership fees of buyers and sellers are

$$\begin{aligned} \frac{\partial \Pi^i}{\partial m_b^i} &= n_b^i + \left(m_b^i - f_b\right)\frac{\partial n_b^i}{\partial m_b^i} + \left(m_s^i - f_s\right)\frac{\partial n_s^i}{\partial m_b^i} = 0 \\ \frac{\partial \Pi^i}{\partial m_s^i} &= \left(m_b^i - f_b\right)\frac{\partial n_b^i}{\partial m_s^i} + n_s^i + \left(m_s^i - f_s\right)\frac{\partial n_s^i}{\partial m_s^i} = 0 \end{aligned} \tag{12}$$

The necessary and sufficient condition for the existence of equilibrium is

$$\frac{-2t_b}{N_b}\left(\Delta_s \pi - \frac{2t_s}{N_s}\right) - \Delta_s u \Delta_b \pi > 0 \quad \text{or} \quad \phi > 0 \tag{13}$$

The above condition comes from the second order stability condition and this inequality is assumed to hold for further analysis. This inequality ensures that the profit function of each platform is concave in membership fees and thus, a unique equilibrium exists. Also, the condition 2.13 ensures that there does not exist any asymmetric equilibrium. Thus, in equilibrium, both the platforms will set the same membership fees for buyers and sellers i.e. $m_b^i = m_b^j = m_b^*$ (say) and $m_s^i = m_s^j = m_s^*$ (say). Buyers' and sellers' participation are equal on both the platforms i.e.

$n_b^i = n_b^j = N_b/2, n_s^i = n_s^j = N_s/2$. The membership fees for buyers and sellers and the equilibrium profit of each platform will be given in Lemma 2.5 . The proof is given in the appendix B. 2 .

Lemma 2.5: If condition 2.13 holds, then in equilibrium, both the pure platforms will set the same membership fees for buyers and sellers and earn equal profit, which are given as follows

$$m_b^* = f_b + t_b - N_s\pi\left(\frac{N_s}{2}\right) \tag{14}$$

$$m_s^* = f_s + t_s - N_b u\left(\frac{N_s}{2}\right) - \frac{N_b N_s}{2}\left(u'\left(\frac{N_s}{2}\right) + \pi'\left(\frac{N_s}{2}\right)\right) \tag{15}$$

$$\Pi^* = \frac{N_b t_b + N_s t_s}{2} - \frac{N_b N_s}{2}\left(u\left(\frac{N_s}{2}\right) + \pi\left(\frac{N_s}{2}\right)\right) - \frac{N_b N_s^2}{2}\left(u'\left(\frac{N_s}{2}\right) + \pi'\left(\frac{N_s}{2}\right)\right) \tag{16}$$

where Π^* represents the equilibrium profit of each pure platform. The equilibrium membership fee for buyers is the standard Hotelling price (marginal cost + transportation cost) adjusted downward by the cross group externalities effect. Apart from the cross group externalities term, one additional term appears in the membership fee of sellers because of sellers' competition. Depending on whether $u'(N_s/2) + \pi'(N_s/2)$ is positive or negative, the membership fee for the seller will adjust downward or upward. This is in line with the finding of Belleflamme and Toulemonde [2016].

4.2 Competition among Platforms when both the Platforms enter into the Sellers' Product Space

When both the platforms enter into the sellers' product space, there are two sources of revenue for each platform. The first is the membership fees paid by buyers and sellers. The second is the profit of the seller owned by the platform. Thus, the profit expression of the platform i when it enters into the sellers' product space is given by

$$\Pi^i = n_b^i\left(m_b^i - f_b\right) + n_s^i\left(m_s^i - f_s\right) + \left(n_b^i\pi\left(n_s^i\right) - m_s^i\right)$$

By using equation 2.11, we can express the profit function of platform i solely in terms of membership fees set by both the platforms i.e. we can express buyers' and sellers' participation as a function of $\left(m_b^i, m_s^i, m_b^j, m_s^j\right)$. Thus, we can write the profit maximization problem of platform i as

$$\max_{m_b^i, m_s^i} \Pi^i\left(m_b^i, m_s^i, m_b^j, m_s^j\right) = n_b^i\left(m_b^i - f_b\right) + n_s^i\left(m_s^i - f_s\right) + \left(n_b^i\pi\left(n_s^i\right) - m_s^i\right)$$

where the buyers' and sellers' participation are a function of $m_b^i, m_s^i, m_b^j, m_s^j$. Now, the first order conditions for membership fees paid by buyers and sellers are

$$\begin{aligned}
\frac{\partial \Pi^i}{\partial m_b^i} &= n_b^i + \left(m_b^i - f_b\right)\frac{\partial n_b^i}{\partial m_b^i} + \left(m_s^i - f_s\right)\frac{\partial n_s^i}{\partial m_b^i} + \pi\left(n_s^i\right)\frac{\partial n_b^i}{\partial m_b^i} + n_b^i\pi'\left(n_s^i\right)\frac{\partial n_s^i}{\partial m_b^i} = 0 \\
\frac{\partial \Pi^i}{\partial m_s^i} &= \left(m_b^i - f_b\right)\frac{\partial n_b^i}{\partial m_s^i} + n_s^i + \left(m_s^i - f_s\right)\frac{\partial n_s^i}{\partial m_s^i} + \pi\left(n_s^i\right)\frac{\partial n_b^i}{\partial m_s^i} + n_b^i\pi'\left(n_s^i\right)\frac{\partial n_s^i}{\partial m_s^i} = 0
\end{aligned} \tag{17}$$

Condition 2.13 will ensure that a unique symmetric equilibrium exists. Thus, in equilibrium, both the platforms will charge the same membership fees, and buyers' and sellers' participation will be equal on both the platforms, i.e $m_b^i = m_b^j = m_b^o$ (say), $m_s^i = m_s^j = m_s^o$ (say) and $n_b^i = n_b^j = N_b/2, n_s^i = n_s^j = N_s/2$. And therefore, both the platforms will enjoy the same profit i.e $\Pi^i = \Pi^j = \Pi^o$ (say). The membership fees for buyers and sellers and the equilibrium profit of each platform will be given in Lemma 2.6. The proof is given in the appendix B.2.

Lemma 2.6: When both the platforms enter into the sellers' product space and condition 2.13 holds; then in equilibrium, both the platforms will set the same membership fees for buyers and sellers and earn equal profit, given by

$$m_b^o = f_b + t_b - N_s\pi\left(\frac{N_s}{2}\right) + \pi\left(\frac{N_s}{2}\right) \tag{18}$$

$$m_s^o = f_s + t_s - N_b u\left(\frac{N_s}{2}\right) - \frac{N_b N_s}{2}\left(u'\left(\frac{N_s}{2}\right) + \pi'\left(\frac{N_s}{2}\right)\right) + \frac{N_b}{2}\pi'\left(\frac{N_s}{2}\right) - \frac{2t_s}{N_s} \tag{19}$$

$$\begin{aligned}
\Pi^o = \frac{N_b t_b + N_s t_s}{2} &- \frac{N_b N_s}{2}\left(u\left(\frac{N_s}{2}\right) + \pi\left(\frac{N_s}{2}\right)\right) - \frac{N_b N_s^2}{2}\left(u'\left(\frac{N_s}{2}\right) + \pi'\left(\frac{N_s}{2}\right)\right) \\
&+ \left[N_b\left(u\left(\frac{N_s}{2}\right) + \pi\left(\frac{N_s}{2}\right)\right) - \frac{N_b}{2}\pi'\left(\frac{N_s}{2}\right)\right) + \frac{2t_s}{N_s}\Bigg] + \\
&\left[\frac{N_b N_s}{2}\left(u'\left(\frac{N_s}{2}\right) + \frac{3}{2}\pi'\left(\frac{N_s}{2}\right)\right) - f_s - 2t_s\right]
\end{aligned} \tag{20}$$

4.3 Comparison of Membership Fees and Platform's Profit

Lemma 2.5 and 2.6 summarizes the membership fees and the platform's profit when both the platforms are pure platforms and when both the platforms enter into the sellers' product space, respectively. By comparing the membership fees and platform's profit under these two scenarios, we will find whether the agent is losing or benefiting when platforms decide to enter into sellers' product space.

By comparing equations 2.14 and 2.18 , we get

$$m_b^o - m_b^* = \pi\left(\frac{N_s}{2}\right)$$

since $\pi() > .0$. Therefore, the membership fee for buyers when the platform enters into the sellers' product space is always higher than the pure platform case i.e. $m_b^o > m_b^*$. Since equilibrium buyers' and sellers' participation are the same in both the scenarios, a higher membership fee for the buyer means loss in the surplus of buyers.

Similarly, on comparing equations 2.15 and 2.15 , we get

$$m_s^o - m_s^* = \frac{N_b}{2}\pi'\left(\frac{N_s}{2}\right) - \frac{2t_s}{N_s}$$

Since $\pi'() < 0, .m_s^o - m_s^* < 0$. Thus, the sellers' membership fee is lower when both the platforms enter the sellers' product space. A lower membership fee leads to higher surplus for sellers.

Again, by comparing equations 2.16 and 2.20 , we get

$$\begin{aligned}\Pi^o - \Pi^* = & \left[N_b\left(u\left(\frac{N_s}{2}\right) + \pi\left(\frac{N_s}{2}\right)\right) - \frac{N_b}{2}\pi'\left(\frac{N_s}{2}\right) + \frac{2t_s}{N_s}\right] \\ & + \left[\frac{N_b N_s}{2}\left(u'\left(\frac{N_s}{2}\right) + \frac{3}{2}\pi'\left(\frac{N_s}{2}\right)\right) - f_s - 2t_s\right].\end{aligned}$$

Since $u'\left(N_s/2\right) + 1.5\pi'\left(N_s/2\right)$ can be positive or negative, it will only be beneficial for both the platforms to enter the sellers' product space if the following condition holds

$$\left[N_b\left(u\left(\frac{N_s}{2}\right) + \pi\left(\frac{N_s}{2}\right)\right) - \frac{N_b}{2}\pi'\left(\frac{N_s}{2}\right)\right) + \frac{2t_s}{N_s}\right] > f_s + 2t_s - \left[\frac{N_b N_s}{2}\left(u'\left(\frac{N_s}{2}\right) + \frac{3}{2}\pi'\left(\frac{N_s}{2}\right)\right)\right] \tag{21}$$

Thus, whenever the condition 2.21 holds, it is profitable for both the platforms to enter into the sellers' product space.

Proposition 2.2: If condition 2.21 holds, then entry of both the platforms into the sellers' product space leads to higher profitability of each platform and higher surplus for sellers. However, surplus for buyers reduces because of entry of platforms into the sellers' product space.

5 Competition Among Platforms: An Alternative Framework

In this section, we again consider a situation where two platforms compete with each other as in the section 2.4, but in a different setup. We assume that each agent will do single homing. As in the

previous sections, here, we compare and contrast the membership fees for buyers and sellers and the profit of each platform under the two scenarios - when both the platforms are pure platforms and when both the platforms enter into sellers' product space.

As in the setup in section 2.4, we assume that there are two platforms, denoted by A and B, which are situated at two extremes of unit interval. There are N_s number of sellers who are uniformly distributed on the unit interval. Now, the differences in this setup from the setup in section 2.4 are as follows. First, we have assumed that the buyers are horizontally and vertically differentiated, where both the differentiations are independent of each other and uniformly distributed on the unit interval [3]. Second, we suppose that the intrinsic utility derived by each agent on the platform will linearly depend on the number of agents of the group on that platform. Let the rate of change of intrinsic utility with respect to participation of the other set of agents be denoted by α_b and α_s for buyers and sellers respectively. Thus, the intrinsic utility derived by each buyer on platform i will be α_b times the number of sellers on platform i. Similarly, for each seller on platform i, intrinsic utility is given by α_s times the buyers' participation on platform i. Third, we presume that, the opportunity cost of buyers and sellers will increase linearly with the distance from the platform at a rate t. The opportunity cost of each buyer is determined solely by the horizontal differentiation. Fourth, we have assumed that buyers will consider that the goods provided by the seller owned by the platform is of high quality. This higher quality is just a perception of each buyer. For example, the good might have been stamped 'verified' by the platform or may be provided with better exchange options within a certain period of time. Let the perceived quality of good provided by the seller which is owned by the platform be denoted by s^o and the quality of goods provided by all the other sellers be denoted by s. Fifth, we suppose that each buyer on the platform will make one and only one transaction.

Let m_b^i and m_s^i denote the membership fees set by the platform i for each buyer and seller respectively. Let the price charged by the seller which is owned by the platform i for the good of quality s^o be p^{io} and the price charged by other sellers on platform i for the good of quality s be p^i. Let n_b^i and n_s^i denote the fraction of total buyers and sellers respectively, who join platform i. Now, when both the platforms enter the sellers' product space, the buyers have two options. Those buyers who join platform i, can either buy the good of quality s at price p^i or buy the good of perceived quality s^o at price p^{io}. Depending upon the taste preferences of each buyer, denoted by θ, they will

[3]Buyers are horizontally differentiated- they interact with one of the two platforms on the basis of their relative locations. Furthermore, by vertical differentiation, I refer to buyers' preference over some quality dimension.

either buy a good of quality s or s°.

When both the platforms are pure platforms, the utility of each buyer and seller who join platform i, denoted by v_b^i and v_s^i respectively, are given by [4]

$$v_b^i(\theta) = \alpha_1 \left[n_s^i \cdot N_s\right] + \left(\theta s - p^i\right) - m_b^i \tag{22}$$

$$v_s^i = \alpha_s \left[n_b^i \cdot N_b\right] + \left(p^i - c\right) \frac{n_b^i N_b}{n_s^i N_s} - m_s^i \tag{23}$$

where c is the marginal cost of the good provided by the sellers.

When both the platforms enter the sellers' product space, each buyer has two options. Therefore, the utility derived by a buyer on platform i will depend on whether the buyer buys goods of quality s or of quality s^o i.e. buy from the seller owned by the platform. The utility of each buyer who joins platform i and buys goods of quality s, is given by equation 2.22. When the buyer on platform i buys a good of quality s^o, its utility is given by

$$v_b^{io}(\theta) = \alpha_1 \left[n_s^i \cdot N_s\right] + \left(\theta s^o - p^{io}\right) - m_b^i \tag{24}$$

A buyer of taste preference θ who joins the platform i will buy goods from the seller who is owned by the platform if only if $v_b^{io}(\theta) > v_b^i(\theta)$. Thus, only those buyers who join platform i and has $\theta \geqslant \frac{p^{io}-p^i}{s^o-s} = \bar{\theta}_i$ (say) are going to buy goods of quality s^o. Let n_b^{io} denote the fraction of buyers who join platform i and buy the good of quality s°. Thus, the number of buyers who join platform i and buy goods of quality s^o is $n_b^{io} N_b$ which is equal to $\left(1 - \bar{\theta}_i\right) n_b^i N_b$ because θ is uniformly distributed on the unit interval.

When both the platforms enter the sellers' product space, the utility of each seller which is not owned by the platform, denoted by v_s^{io}, is given by

$$v_s^{io} = \alpha_s \left[n_b^i \cdot N_b\right] + \left(p^i - c\right) \frac{\bar{\theta}_i n_b^i N_b}{n_s^i N_s} - m_s^i \tag{25}$$

The stages of the game are as follows

[4]The utilities in these formulae capture a setup wherein one set of agents benefits from a larger presence of the opposite set of agents on one platform vis-a-vis the other. For instance, customers who are looking for a cab ride (i.e, buyers) may care about the presence of a larger number of drivers (i.e, sellers) on Uber (platform 1) in comparison to Ola (platform 2).

Stage 1 : Platforms set membership fees for both the groups i.e. $\left(m_b^i, m_s^i\right)$ and $\left(m_b^j, m_s^j\right)$, simultaneously and independently, which are observable and verifiable by all.

Stage 2 : After observing the membership fees of both the platforms, buyers and sellers simultaneously decide which platform to join.

Stage 3 : Sellers on both the platforms engage in price competition and decide the good's price i.e. p^i and p^j. In the second scenario, price competition decides p^i and p^{io} on platform i.

We solve the game by backward induction. Whenever the number of sellers on each platform is more than or equal to three [5], price competition implies that each seller which is not owned by the platform will set price equal to marginal cost. The following assumption ensures this minimum sellers' participation on each platform.

Assumption 2.1 The sellers' participation on each platform is more than or equal to 3 i.e $n_s^i N_s \geqslant 3$ where $i \in \{A, B\}$.

5.1 Competition Among Pure Platforms

When both the platforms are pure platforms, the only sources of revenues are membership fees charged from buyers and sellers. Let f_b and f_s denote the cost of serving each buyer and seller respectively. Then the profit of pure platform i is given by

$$\pi^i = n_b^i N_b \left(m_b^i - f_b\right) + n_s^i N_s \left(m_s^i - f_s\right)$$

Solution of Stage 3:

If assumption 2.1 holds, then price competition among sellers on both the pure platforms will result in the Bertrand outcome i.e. $p^i = c$ where $i \in \{A, B\}$.

Solution of Stage 2:

When both the platforms are pure platforms, there is no differentiation in quality among the goods available on each platform. Let the horizontal position of the buyer who is indifferent between joining platform i or j be situated at a distance $\tilde{x}$ from platform i. Then, all the buyers who are to the left of $\tilde{x}$ will join platform i. The buyer at $\tilde{x}$ will get the same utility by joining platform i or j.

[5] In case of pure platforms, even when the number of sellers on each platform is greater than or equal to 2, we get price equal to marginal cost as a result of price competition. But when both the platforms enter the sellers' product space, a minimum of 3 sellers are required on each platform to get price equal to marginal cost.

$$v_b^i(\theta) - t(\tilde{x}) = v_b^j(\theta) - t(1-\tilde{x})$$
$$\tilde{x} = \frac{1}{2} + \frac{v_b^i(\theta) - v_b^j(\theta)}{2t}$$

Using equation 2.22 in the above expression, we get

$$n_b^i = \tilde{x} = \frac{1}{2} + \frac{\alpha_1 N_s \left(2n_s^i - 1\right) + \left(m_b^j - m_b^i\right)}{2t} \tag{26}$$

Similarly, let the seller at $\tilde{y}$ be indifferent between joining platform i and platform j i.e. the utility derived by the seller at $\tilde{y}$ by joining platform i or j will be the same.

$$v_s^i - t(\tilde{y}) = v_s^j - t(1-\tilde{y})$$
$$\tilde{y} = \frac{1}{2} + \frac{v_s^i - v_s^j}{2t}$$

Using equation 2.23 in the above expression, we get

$$n_s^i = \tilde{y} = \frac{1}{2} + \frac{\alpha_s N_b \left(2n_b^i - 1\right) + \left(m_s^j - m_s^i\right)}{2t} \tag{27}$$

The market structure in competition among pure platforms is as shown in figure 2.1.

By solving equations 2.26 and 2.27 simultaneously, we get

$$n_b^i = \frac{1}{2} + \frac{\alpha_b N_s \left(m_s^j - m_s^i\right) + t\left(m_b^j - m_b^i\right)}{2\left(t^2 - \alpha_b \alpha_s N_s N_b\right)} \tag{28}$$
$$n_s^i = \frac{1}{2} + \frac{\alpha_s N_b \left(m_b^j - m_b^i\right) + t\left(m_s^j - m_s^i\right)}{2\left(t^2 - \alpha_b \alpha_s N_s N_b\right)} \tag{29}$$

Solution of Stage 1:

In stage 1, both the pure platforms set membership fees for both the groups simultaneously and independently i.e. platform i and platform j decide $\left(m_b^i, m_s^i\right)$ and $\left(m_b^j, m_s^j\right)$ simultaneously such that it maximizes their overall profit. The profit expression of platform i is given by

$$\pi^i = \left(n_b^i N_b\right) m_b^i + \left(n_s^i N_s\right) m_s^i$$

Now the first order condition for platform i will be given by

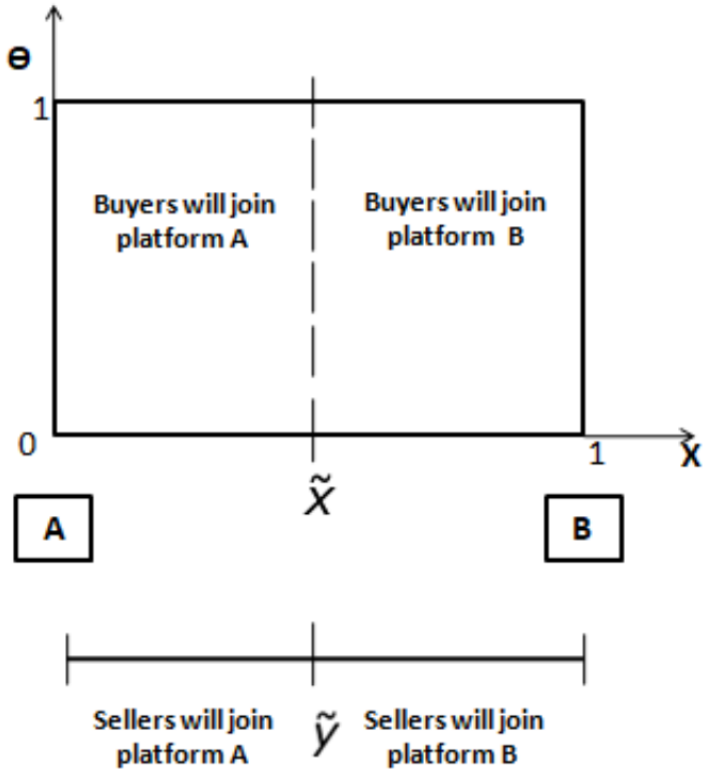


Figure 1: Market Structure in Competition among Pure Platforms

$$\frac{\partial \pi^i}{\partial m_b^i} = N_b \left[n_b^i + m_b^i \cdot \frac{\partial n_b^i}{\partial m_b^i} \right] + N_s \left[m_s^i \cdot \frac{\partial n_s^i}{\partial m_b^i} \right] = 0 \tag{30}$$

$$\frac{\partial \pi^i}{\partial m_s^i} = N_b \left[m_b^i \cdot \frac{\partial n_b^i}{\partial m_s^i} \right] + N_s \left[n_s^i + m_s^i \cdot \frac{\partial n_s^i}{\partial m_s^i} \right] = 0 \tag{31}$$

From equations 2.28 and 2.29 , we know

$$\frac{\partial n_b^i}{\partial m_b^i} = \frac{-t}{2\left(t^2 - \alpha_b \alpha_s N_s N_b\right)}; \quad \frac{\partial n_s^i}{\partial m_b^i} = \frac{-\alpha_s N_b}{2\left(t^2 - \alpha_b \alpha_s N_s N_b\right)}$$

$$\frac{\partial n_b^i}{\partial m_s^i} = \frac{-\alpha_b N_s}{2\left(t^2 - \alpha_b \alpha_s N_s N_b\right)}; \quad \frac{\partial n_s^i}{\partial m_s^i} = \frac{-t}{2\left(t^2 - \alpha_b \alpha_s N_s N_b\right)}$$

The necessary and sufficient conditions for the existence of an equilibrium are

$$\frac{\partial^2 \pi^i}{\partial \left(m_b^i\right)^2} = 2\frac{\partial n_b^i}{\partial m_b^i} < 0; \quad \frac{\partial^2 \pi^i}{\partial \left(m_s^i\right)^2} = 2\frac{\partial n_s^i}{\partial m_s^i} < 0$$

And

$$\begin{vmatrix} \frac{\partial^2 \pi^i}{\partial (m_b^i} & \frac{\partial^2 \pi^i}{\partial m_b^i \partial m_s^i} \\ \frac{\partial^2 \pi^i}{\partial m_s^i \partial m_b^i} & \frac{\partial^2 \pi^i}{\partial (m_s^i)^2} \end{vmatrix} > 0$$

On solving the above expression, we get

$$4t^2 - (\alpha_b + \alpha_s)^2 N_b N_s > 0 \tag{32}$$

Condition 2.32 ensures that the profit function of each platform is concave in membership fees and thus, a unique equilibrium exists. Also, condition 2.32 ensures that there does not exist any asymmetric equilibrium. Thus in equilibrium, both the pure platforms will set the same membership fees for buyers and for sellers i.e. $m_b^i = m_b^j = m_b^*$ (say) and $m_s^i = m_s^j = m_s^*$.

Buyers' and sellers' participation are equal on both the platforms. Now, equations 2.30 and 2.31 will reduce to

$$(t)m_b^* + (\alpha_s N_s)\, m_s^* = t^2 - \alpha_b \alpha_s N_s N_b$$

$$(\alpha_b N_b)\, m_b^* + (t)m_s^* = t^2 - \alpha_b \alpha_s N_s N_b$$

On solving the above equations simultaneously, we get the equilibrium membership fees for buyers and sellers. The equilibrium membership fee for buyers and sellers and the profit of each platform are given in Lemma 2.7.

Lemma 2.7 Suppose that the condition 2.32 holds, then in equilibrium, both the pure platforms will set the same membership fees for buyers and sellers and earn equal profit, given by

$$m_b^* = t - \alpha_s N_s \tag{33}$$

$$m_s^* = t - \alpha_b N_b \tag{34}$$

$$\pi^* = \frac{t\,(N_b + N_s)}{2} - \frac{(\alpha_b + \alpha_s)\, N_b N_s}{2} \tag{35}$$

where π^* represents the equilibrium profit of each pure platform.

5.2 Competition Among Platforms When Both the Platforms Enter the Sellers' Product Space

When both the platforms enter the sellers' product space, there are two sources of revenue for each platform. The first is the membership fees of buyers and sellers and the second, the profit of the

seller owned by the platform. Thus, the profit expression of platform i when it enters the sellers' product space is given by

$$\pi^i = \left(n_b^i N_b\right) m_b^i + \left(n_s^i N_s\right) m_s^i + \left(n_b^i N_b\right)\left(1 - \bar{\theta}_i\right)\left(p^{io} - c\right) - m_s^i$$

where $\left(n_b^i N_b\right)\left(1 - \bar{\theta}_i\right)$ is the fraction of buyers who join platform i and buy from the seller which is owned by the platform i.

Depending upon the taste preferences of the buyers, they either choose to buy from the seller which is owned by the platform or from the other sellers available on the platform. If a buyer on platform i has a taste preference less than $\bar{\theta}_i$, then he will choose to buy the good of quality s and get a utility given by equation 2.22 . Otherwise, he will choose to buy the good of quality s^o and get a utility given by equation 2.24 . The utility of each seller on platform i is given by equation 2.25 .

Solution of stage 3:

Suppose that assumption 2.1 holds. Then, price competition among the sellers which are not owned by the platform will result in the Bertrand outcome i.e. $p^i = c$. The platform i will decide p^{io} such that it maximizes the overall profit in stage 3 . The profit maximization problem of platform i in stage 3 for given $\left(m_b^i, m_s^i\right)$ and $\left(n_b^i, n_s^i\right)$ is as follows

$$\max_{p^{io}} \pi^i = \left(n_b^i N_b\right) m_b^i + \left(n_s^i N_s - 1\right) m_s^i + \left(n_b^i N_b\right)\left(1 - \bar{\theta}_i\right)\left(p^{io} - c\right)$$

where $\bar{\theta}_i = \frac{p^{io} - p^i}{s^o - s}$ From first order condition, we get

$$\bar{\theta}_i = \frac{1}{2}$$

That is

$$p^{io} = c + \frac{s^o - s}{2} \quad \text{where} \quad i \in \{A, B\}$$

$p^{Ao} = p^{Bo} = p^o$ (say). The distribution of buyers when both the platforms enter the sellers' product space will be as shown in figure 2.2 .

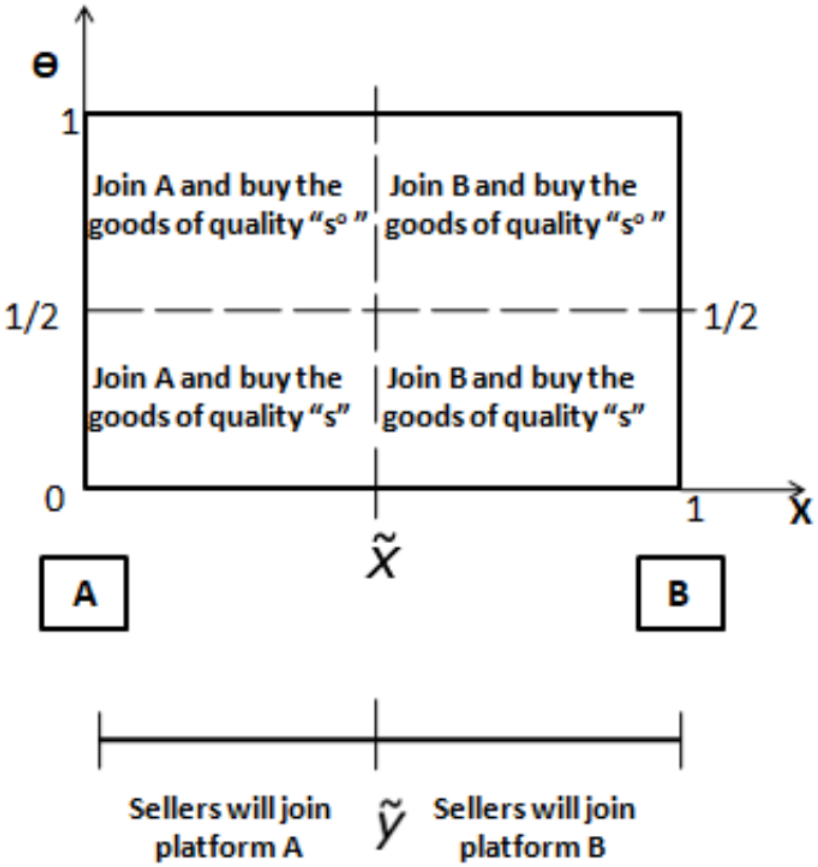


Figure 2: Market Structure When Both the Platforms Enter the Sellers' Product Space

Solution of stage 2:

Again, assume that the buyer and the seller who are indifferent between joining platform i and j are located at $\tilde{x}\tilde{y}$ respectively. As calculated in the case of the pure platform, we get

$$\tilde{x} = n_b^i = \frac{1}{2} + \frac{\alpha_b N_s \left(m_s^j - m_s^i\right) + t\left(m_b^j - m_b^i\right)}{2\left(t^2 - \alpha_b \alpha_s N_s N_b\right)}$$

$$\tilde{y} = n_s^i = \frac{1}{2} + \frac{\alpha_s N_b \left(m_b^j - m_b^i\right) + t\left(m_s^j - m_s^i\right)}{2\left(t^2 - \alpha_b \alpha_s N_s N_b\right)}$$

with $n_b^{io} = \frac{n_b^i}{2}$

Solution of stage 1:

In stage 1, both the platforms set membership fees for buyers and sellers simultaneously and independently. The profit maximization problem for platform i in stage 1 is given by

$$\max_{m_b^i, m_s^i} \pi^i = \left(n_b^i N_b\right) m_b^i + \left(n_s^i N_s - 1\right) m_s^i + \frac{\left(n_b^i N_b\right)}{2} \left(p^o - c\right)$$

First order conditions given by $\frac{\partial \pi^i}{\partial m_b^2} = 0$; $\frac{\partial \pi^i}{\partial m_s^i} = 0$ will decide the equilibrium membership fees for buyers and sellers. Condition 2.32 ensures that a unique symmetric equilibrium exists. Thus, in equilibrium, both the platforms will charge the same membership fees and buyers' and sellers' participation will be equal on both the platforms. That is, in equilibrium, $m_b^i = m_b^j = m_b^o$ (say) and $m_s^i = m_s^j = m_s^o$ (say); and $n_b^i = n_b^j = \frac{1}{2}$ and $n_b^i = n_b^j = \frac{1}{2}$. Now, the first order condition will reduce to

$$(t) m_b^o + \left(\alpha_s N_s\right) m_s^o = t^2 - \alpha_b \alpha_s N_s N_b - \frac{\left(p^o - c\right) \cdot t}{2}$$

$$\left(\alpha_b N_s N_b\right) m_b^o + \left(t N_s\right) m_s^o = t^2 - \alpha_b \alpha_s N_s N_b \left(N_s - 2\right) - \frac{\left(p^o - c\right) \alpha_b N_b N_s}{2}$$

On solving the above equations simultaneously, we get equilibrium membership fees for buyers and sellers. The equilibrium membership fees for buyers and sellers and the profit of each platform are as in Lemma 2.8.

Lemma 2.8: When both the platforms enter the sellers' product space and given condition 2.32 holds, then in equilibrium, both the platforms will set the same membership fees for buyers and sellers and earn equal profit given by

$$m_b^o = t - \alpha_s N_s + 2\alpha_s - \frac{\left(p^o - c\right)}{2} \tag{36}$$

$$m_s^o = t - \alpha_b N_b - \frac{2t}{N_s} \tag{37}$$

$$\pi^o = \frac{t\left(N_b + N_s\right)}{2} - \frac{\left(\alpha_b + \alpha_s\right) N_b N_s}{2} + \left(\alpha_s N_b - t\right) \tag{38}$$

where π^o represents the equilibrium profit of each platform when both the platforms enter the sellers' product space.

5.3 Comparison of Membership Fees and Platform's Profit

Lemma 2.7 and 2.8 summarize membership fees and platform's profit when both the platforms are pure platforms and when both the platforms enter the sellers' product space, respectively. By

comparing membership fees and platform's profit under these two scenarios, we examine whether an agent is losing or benefitting when platforms decide to enter the sellers' product space.

By comparing equations 2.33 and 2.36 , we get

$$m_b^o - m_b^* = 2\alpha_s - \frac{p^o - c}{2}$$

Thus,

$$m_b^o - m_b^* < 0 \quad \Rightarrow \quad 2\alpha_s - \frac{p^o - c}{2} < 0, \quad \text{or} \quad \alpha_s < \frac{s^o - s}{8} \tag{39}$$

When condition 2.39 holds, equilibrium membership fees for the buyer is lower in the second scenario. Condition 2.39 means that if the buyer perceives that the quality of goods sold by the seller owned by the platform is sufficiently high, only then the membership fee for the buyer is lower in the second scenario.

Similarly, on comparing equations 2.34 and 2.37 , we get

$$m_s^o - m_s^* = \frac{-2t}{N_s}$$

Since t and N_s both are positive, it follows that $m_s^o - m_s^* < 0$. Thus, the sellers' membership fee is lower when both the platforms enter into the sellers' product space.

Again, by comparing equations 2.35 and 2.38 , we get

$$\Pi^o - \Pi^* = \alpha_s N_b - t$$

Since $\alpha_s N_b - t$ can be positive or negative, it will only be beneficial for both the platforms to enter the sellers' product space if the following condition holds

$$\alpha_s N_b - t > 0, \quad \text{or} \quad \alpha_s > \frac{t}{N_b} \tag{40}$$

Thus, whenever the condition 2.40 holds, it is profitable for both the platforms to enter the sellers' product space.

Proposition 2.3 Suppose that the condition 2.32 holds. Then, the entry of both the platforms into the sellers' product space leads to

1. lower membership fee and consequently higher surplus for sellers;

2. if $\alpha_s < \frac{s^0 - s}{8}$, then lower membership fee and consequently higher surplus for buyers; and

3. if $\alpha_s > \frac{t}{N_b}$, then higher profit for each platform,

compared to that in the case of competition between pure platforms.

From proposition 2.3, when $\frac{t}{N_b} < \alpha_s < \frac{s^o - s}{8}$, entry of both the platforms into the sellers' product space will lead to higher surplus accruing to buyers and higher profit for each platform. So, when per buyer intrinsic utility of each seller lies between $\frac{t}{N_b}$ and $\frac{s^o - s}{8}$, then both buyers and platforms are better off when platforms enter the sellers' product space compared to the scenario with pure platforms.

Since sellers always benefit from the entry of platforms into sellers' product space, when $\frac{t}{N_b} < \alpha_s < \frac{s^o - s}{8}$, it is a win-win situation for buyers, sellers and platforms when platforms enter into the sellers' product space, in comparison to the case in which they remain pure platforms.

6 Conclusion

This chapter compares and contrasts buyers' and sellers' surplus and the profit generated by the platform under two alternative scenarios. In the first scenario, the platform(s) engages in connecting buyers and sellers only whereas in the second scenario, the platform(s) apart from connecting buyers and sellers, enters into sellers' product space and start competing with other sellers on the platform.

In the case of a monopoly platform, the entry of the platform into the sellers' product space has two effects. First, it leads to a decrease in the membership fee for sellers and thus higher sellers' participation on the platform. This intensifies the competition among sellers. Second, it increases the consumer base which in turn increases the demand for each seller. An increase in the number of sellers decreases the per buyer profit for each seller while an increase in the consumer base will have a positive effect on the profit for each seller. Hence, the overall effect of the entry of a monopoly platform into the sellers' product space will depend on the relative magnitudes of these two opposing effects. The overall effect turns out to be beneficial for sellers. Also, the decrease in membership fee for the sellers nullify the effect of increased competition among the sellers. In equilibrium, under some restrictions, the entry of the monopoly platform into the sellers' product space is beneficial for all - buyers, sellers, and the platform in comparison to the pure monopoly platform case.

Two frameworks for competition among platforms have been developed in this chapter. In the first framework, buyers are only horizontally differentiated as in the case of a monopoly platform. In this framework, the entry of both the platforms into the sellers' product space is beneficial for sellers but the buyers will lose out, teh benchmark being the pure platform case. In the second

framework, buyers are horizontally and vertically differentiated and perceive the good sold by the platform to be of high quality. In this framework, the entry of platforms into sellers' product space is beneficial for all - buyers, sellers, and the platforms, under some restrictions. Thus, in the case of competition among platforms, entry of both the platforms into sellers' product space is always beneficial for sellers compared to the scenario in which of them are pure platforms. However, the buyers either lose out or are better off under some restrictions, depending upon the framework of competition.

Policy Discussion: On December 26th 2018, Ministry of Commerce & Industry, Government of India released a circular on the policy on Foreign Direct Investment in e-commerce which restricts e-commerce marketplace from having an equity in the company which operates as a seller on the same platform (Dept of Industrial Policy and Promotion, 2018) [6]. That is, those sellers which are owned (either fully or partly) by the platform are not allowed to operate as a seller on the same platform. However, the analysis in this chapter demonstrates that such a policy that restrict platforms' entry into the sellers' product space will not only harm the platform and the buyers but also the sellers since it will decrease the consumer base and increase the membership fee for the seller. Also, there is some empirical evidence which shows that many a time, platforms enter into the sellers' segment to improve the quality of the platform which reduces the search cost for the buyers and in turn, increases the demand for good sellers. According to Hagiu and Spulber [2013], platforms solve the coordination problem in a two sided market by providing first party content. Now, if platforms are not allowed to enter into sellers' product space, it will become tougher for the platform to solve the coordination problem, especially for a new platform. Thus, the above mentioned policy seems to have a detrimental effect on the prospect of entry of a platform as well.

[6] see Press Note No. 2

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7 Appendix A

7.1 Proof of Lemma 2.2

When the seller owned by the platform has an outside option higher than the intrinsic utility of joining the platform, the platform's profit is higher in the second scenario for any given buyer and seller's participation. That is, $\Pi^o \geq \Pi$ for any given pair of (n_b, n_s), provided the $z_s \geq r_s$. Therefore

$$\Pi^o|_{(n_b^*, n_s^*)} \geq \Pi|_{(n_b^*, n_s^*)} \quad \text{provided} \quad z_s \geq r_s \tag{41}$$

where n_b^* and n_s^* are the equilibrium buyers and sellers participation when the monopoly platform is a pure platform. Also, since the equilibrium buyers and sellers' participation is unique, therefore

$$\Pi^o|_{(n_b^o, n_s^o)} > \Pi^o|_{(n_b^*, n_s^*)} \tag{42}$$

This is because (n_b^o, n_s^o) are the buyers' and sellers' participation at which, the profit of the monopoly platform maximizes in the second scenario. Thus, the profit at any other buyer and seller participation will always be lower. Now, from conditions 41 and 42, we get

$$\Pi^o|_{\left(n_b^o, n_s^o\right)} > \Pi|_{\left(n_b^*, n_s^*\right)} \quad \text{provided} \quad z_s \geq r_s$$

7.2 Proof of Lemma 2.5 and Lemma 2.6

Differentiating equation 2.11 with respect to m_b^i, we get

$$\begin{aligned} \tfrac{-2t_b}{N_b} \tfrac{\partial n_b^i}{\partial m_b^i} + \Delta_s u\left(n_s^i\right) \tfrac{\partial n_s^i}{\partial m_b^i} = 1; \\ \Delta_b \pi\left(n_b^i, n_s^i\right) \tfrac{\partial n_b^i}{\partial m_b^i} + \Delta_s \pi\left(n_b^i, n_s^i\right) - \tfrac{2t_s}{N_s} \tfrac{\partial n_s^i}{\partial m_b^i} = 0 \end{aligned} \tag{43}$$

Solve equations 43 simultaneously, we get

$$\frac{\partial n_b^i}{\partial m_b^i} = \frac{\Delta_s \pi\left(n_b^i, n_s^i\right) - 2t_s/N_s}{\phi}; \quad \frac{\partial n_s^i}{\partial m_b^i} = \frac{-\Delta_b \pi\left(n_b^i, n_s^i\right)}{\phi} \tag{44}$$

Similarly, by differentiating equation 2.11 by m_s^i, and solving equations simultaneously, we get

$$\frac{\partial n_b^i}{\partial m_s^i} = \frac{-\Delta_s u\left(n_s^i\right)}{\phi}; \quad \frac{\partial n_s^i}{\partial m_s^i} = \frac{-2t_b/N_b}{\phi} \tag{45}$$

$$\begin{aligned} \Delta_s u\left(\frac{N_s}{2}\right) &= 2u\left(\frac{N_s}{2}\right) + N_s u'\left(\frac{N_s}{2}\right) \\ \Delta_b \pi\left(\frac{N_b}{2}, \frac{N_s}{2}\right) &= 2\pi\left(\frac{N_s}{2}\right) \\ \Delta_s \pi\left(\frac{N_b}{2}, \frac{N_s}{2}\right) &= N_b \pi'\left(\frac{N_s}{2}\right) \end{aligned} \tag{46}$$

Also, we know

7.3 Proof of Lemma 2.5

When both the platforms are pure platforms, first order conditions for membership fee are given equation 2.12 and are as follows

$$\begin{aligned} \frac{\partial \Pi^i}{\partial m_b^i} &= n_b^i + \left(m_b^i - f_b\right) \frac{\partial n_b^i}{\partial m_b^i} + \left(m_s^i - f_s\right) \frac{\partial n_s^i}{\partial m_b^i} = 0 \\ \frac{\partial \Pi^i}{\partial m_s^i} &= \left(m_b^i - f_b\right) \frac{\partial n_b^i}{\partial m_s^i} + n_s^i + \left(m_s^i - f_s\right) \frac{\partial n_s^i}{\partial m_s^i} = 0 \end{aligned}$$

Equilibrium is symmetric, so the membership fees and buyers' and sellers' participation are equal on both platforms. Now, by using equations 44 and 45 in the above first-order conditions, we get

$$\begin{aligned}\left(\Delta_s\pi - \tfrac{2t_s}{N_s}\right)(m_b^* - f_b) - (\Delta_b\pi)(m_s^* - f_s) &= -\tfrac{N_b}{2}\phi \\ (-\Delta_s u)(m_b^* - f_b) - \left(\tfrac{2t_b}{N_b}\right)(m_s^* - f_s) &= -\tfrac{N_s}{2}\phi\end{aligned} \tag{47}$$

By solving equations 47 simultaneously, we get

$$\begin{aligned}m_b^* &= f_b + t_b - \frac{N_s}{2}\Delta_b\pi \\ m_s^* &= f_s + t_s - \frac{N_b}{2}\Delta_s u - \frac{N_s}{2}(\Delta_s\pi)\end{aligned}$$

Now by using equation 46 in the expression of m_b^* and m_s^*, we get

$$\begin{aligned}m_b^* &= f_b + t_b - N_s\pi\left(\frac{N_s}{2}\right) \\ m_s^* &= f_s + t_s - N_b u\left(\frac{N_s}{2}\right) - \frac{N_b N_s}{2}\left(u'\left(\frac{N_s}{2}\right) + \pi'\left(\frac{N_s}{2}\right)\right)\end{aligned}$$

By using these membership fees in the profit expression, we get

$$\Pi^* = \frac{N_b t_b + N_s t_s}{2} - \frac{N_b N_s}{2}\left(u\left(\frac{N_s}{2}\right) + \pi\left(\frac{N_s}{2}\right)\right) - \frac{N_b N_s^2}{2}\left(u'\left(\frac{N_s}{2}\right) + \pi'\left(\frac{N_s}{2}\right)\right)$$

7.4 Proof of Lemma 2.6

When both the platforms enter into the sellers' product space, first order conditions for membership fee are given equation 2.17 , and are as follow

$$\begin{aligned}\frac{\partial\Pi^i}{\partial m_b^i} &= n_b^i + \left(m_b^i - f_b\right)\frac{\partial n_b^i}{\partial m_b^i} + \left(m_s^i - f_s\right)\frac{\partial n_s^i}{\partial m_b^i} + \pi\left(n_s^i\right)\frac{\partial n_b^i}{\partial m_b^i} + n_b^i\pi'\left(n_s^i\right)\frac{\partial n_s^i}{\partial m_b^i} = 0 \\ \frac{\partial\Pi^i}{\partial m_s^i} &= \left(m_b^i - f_b\right)\frac{\partial n_b^i}{\partial m_s^i} + n_s^i + \left(m_s^i - f_s\right)\frac{\partial n_s^i}{\partial m_s^i} + \pi\left(n_s^i\right)\frac{\partial n_b^i}{\partial m_s^i} + n_b^i\pi'\left(n_s^i\right)\frac{\partial n_s^i}{\partial m_s^i} = 0\end{aligned}$$

Equilibrium is symmetric, so the membership fees and buyers' and sellers' participation are equal on both platforms. Now, by using equations 44 and 45 in the above first-order conditions, we get

$$\begin{aligned}\left(\Delta_s\pi - \frac{2t_s}{N_s}\right)(m_b^o - f_b) - (\Delta_b\pi)(m_s^o - f_s) &= -\frac{N_b}{2}\phi - \left(\Delta_s\pi - \frac{2t_s}{N_s}\right)\pi\left(\frac{N_s}{2}\right) + \frac{N_b}{2}(\Delta_b\pi)\,\pi'\left(\frac{N_s}{2}\right) \\ (-\Delta_s u)(m_b^o - f_b) - \left(\frac{2t_b}{N_b}\right)(m_s^o - f_s) &= -\frac{N_s}{2}\phi + \phi + (\Delta_s u)\,\pi\left(\frac{N_s}{2}\right) + \frac{N_b}{2}\left(\frac{2t_b}{N_b}\right)\pi'\left(\frac{N_s}{2}\right)\end{aligned} \tag{48}$$

By solving equation 48 simultaneously, we get

$$m_b^o = f_b + t_b - \pi\left(\frac{N_s}{2}\right) + \left(\frac{N_s}{2} - 1\right)\Delta_b\pi$$
$$m_s^o = f_s + t_s - \frac{N_b}{2}\Delta_s u - \frac{N_s}{2}\left(\Delta_s\pi\right) + \left(\Delta_s\pi - \frac{2t_s}{N_s}\right) - \frac{N_b}{2}\pi'\left(\frac{N_s}{2}\right)$$

Now by using equation 46 in the expression of m_b^o and m_s^o, we get

$$m_b^o = f_b + t_b - N_s\pi\left(\frac{N_s}{2}\right) + \pi\left(\frac{N_s}{2}\right)$$
$$m_s^o = f_s + t_s - N_b u\left(\frac{N_s}{2}\right) - \frac{N_b N_s}{2}\left(u'\left(\frac{N_s}{2}\right) + \pi'\left(\frac{N_s}{2}\right)\right) + \frac{N_b}{2}\pi'\left(\frac{N_s}{2}\right) - \frac{2t_s}{N_s}$$

By using these membership fees in the profit expression, we get

$$\Pi^o = \frac{N_b t_b + N_s t_s}{2} - \frac{N_b N_s}{2}\left(u\left(\frac{N_s}{2}\right) + \pi\left(\frac{N_s}{2}\right)\right) - \frac{N_b N_s^2}{2}\left(u'\left(\frac{N_s}{2}\right) + \pi'\left(\frac{N_s}{2}\right)\right)$$
$$+ \left[N_b\left(u\left(\frac{N_s}{2}\right) + \pi\left(\frac{N_s}{2}\right)\right) - \frac{N_b}{2}\pi'\left(\frac{N_s}{2}\right)\right) + \frac{2t_s}{N_s}\Bigg] +$$
$$\left[\frac{N_b N_s}{2}\left(u'\left(\frac{N_s}{2}\right) + \frac{3}{2}\pi'\left(\frac{N_s}{2}\right)\right) - f_s - 2t_s\right]$$